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Simple construction surveying for rural applications

Jan H. Loedeman
Foreword

In autumn 1996 Agromisa asked me to act as a mediator in finding a qualified author for revision of the first version of this booklet, published in 1990. The request meant an open invitation because I felt challenged to include some of my own ideas about the kind of surveying a new edition of Agrodok 6 should address. As usual, a pertinent difference manifested itself between the inception of an idea and its realisation. Several sources of inspiration kept me going.

The unflagging encouragement I have continually received from Agromisa’s publications manager Marg Leijdens and her successor, Margriet Berkhout, has been decisive. I’m grateful for their unrelenting confidence. I tender my thanks to Johan Boesjes, president of GITC bv, who financially enabled the proof-reading of my texts. Without the instantaneous dedication of Kate Ashton this would never have succeeded in time. Kate also served as my voluntary reference reader. She assured me that this Agrodok makes surveying open to the non-professional. Being a non-surveyor by profession myself, I was greatly relieved when my colleague and friend Marc Chieves – licensed surveyor in the USA and currently editor of *The American Surveyor*– assured me that my treatise on the subject was sound. The one-liner with which he communicated his opinion to me will not be easily forgotten.

A vital source of inspiration for this booklet dates back to 1972, to my stay in the Khroumir region of Northwest Tunisia. There I had to swallow some hard lessons from subsistence farmers. As I lived amongst them for five months they gradually made me understand that important aspects of subsistence farming practices do not lend themselves to measurement in the most literal sense. Nevertheless, they showed a lot of interest in the surveying instrument I used to assess the size of their fields in hectares, a quantity that they didn’t need at all. In their turn, these illiterate but skilled and clever men gradually grasped the intellectual power of measurements in combination with models and calculations. The realisation of this booklet is a humble tribute to them and to their current colleagues elsewhere.
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1 Introduction: scope & structure

The majority of farmers in the world still practise some form of subsistence farming. Their draught-animal and handwork-based farming practices, however, cannot be compared with the completely mechanised and highly automated precision-farming practices that are becoming the norm for many of their North American colleagues. Hence, ‘farming’ is a term too general to be explanatory.

The same may be said of the term ‘surveying’. A modern surveyor cannot anymore do without a digital computer, which he needs to swiftly perform complex mathematical operations on measurement data he acquires with sophisticated and highly automated equipment. The same technology enables an earth-moving machine automatically to dig a canal or to terrace a slope, according to the spatial form that has been geometrically designed in a computer and transferred to the machine’s navigation and operating system. But like all farming, surveying too is based on some generic concepts independent of the technology used to put these into practice.

For whom Agrodok 6 is (not) meant

This booklet is written for those who have an interest, for whatever reason, in measurement techniques related to ‘construction & building’ that go beyond the ones a carpenter applies. He or she is supposed to have at least some idea of basic geometric principles. Though factual knowledge of the branch of mathematics called ‘plane geometry’ is not a prerequisite, it will ease the grasping of most subjects presented.

This booklet is definitely not written as a manual to satisfy those who expect or need detailed instructions, presented in cookbook style. Though some ‘recipes’ are included where this is appropriate or clarifying, too much is left open to the imagination of the reader to make this book a comprehensive surveying manual. It is neither meant as that, nor as an aid for drilling surveyors. The purpose is to help people to understand some basic principles that lie at the heart of surveying in general.
What Agrodok 6 is (not) about
This booklet presents surveying in a way it is not practised by any single professional surveyor. This sounds both cryptic and unrealistic, but the opposite is true. Getting to grips with surveying is not so much a matter of learning what a surveyor does, but of what he thinks. Historically, surveying has been characterised by a high degree of labour specialisation. This is reflected in the tasks surveyors perform and the education they receive at various professional levels.
In large construction works those who have received the lowest level of education perform most measurements. Some basic and crucial concepts need not to be explained or known at this level because, within the context of labour distribution, these concepts are addressed at a higher level in the organisation, at which the surveying process is monitored. Many textbooks that deal specifically with ‘simple’ surveying techniques may therefore be of little help in providing an overview of these concepts and that process. On the other hand, at the higher level of education, surveying is dealt with from a mathematical starting point. At this level too methods and techniques are presented and discussed one by one, without surveying being explicitly addressed, from head to tail, as a process.
A more generic problem related to any introductory presentation of surveying lies in the linking of two quite different ‘worlds’. What the surveyor is doing, for instance at a construction-site, is clearly visible and not very different from what the carpenter and the builder are doing there: he takes measures with a device. These actions form the ‘real world’ aspect of surveying. The connection between individual measurements and the cohesion of a surveyor’s actions rely on an abstract world that obeys the laws of geometry and other branches of mathematics.
Geometric models are at the heart of all surveying. Generic surveying problems and their solutions therefore require a transition from the real world (wherein the measurements take place) to the abstract world of geometric models (wherein acquired measurement data are actually used and mutually related). The results of mathematical operations have to be migrated back to the real world again, either on the site or on a piece of paper. From a practical point of view, surveying is
actually to a large extent a matter of on-the-job training. In consequence, the distinction between the real world and the abstract world becomes easily blurred.

This booklet has been conceived as an attempt to present surveying in a generic form by applying geometric concepts without using mathematics. Though abstractions are deliberately not shunned, the presented line of thought is definitely a practical one. Construction surveying offers a very practical context within which to illustrate what surveying is about. The qualification ‘simple’ has been included in the title to express the idea that the technological level of measurement techniques presented is ‘intelligible’ and ‘understandable’. This qualification certainly does not imply an ‘unsophisticated’ or ‘naive’ approach to underlying surveying concepts.

What Agrodok 6 contains and how it is structured

The best way to learn surveying is to be trained and drilled on the job by a professional surveyor. Surveying is like riding a horse or a camel: one will never get to grips with it by merely studying a book about the subject. And, as with so many crafts, it needs a lot of practice. Moreover, some pitfalls and hurdles cannot be addressed on paper at all, like the reconnaissance of a construction-site that needs to be surveyed. Other aspects that require practice are, for instance, keeping clear and tidy field notes, and the level of detail that has to be provided by the survey in relation to a specific construction.

Chapter 2 discusses what comprises construction surveying (Sec. 2.1). The principal aim of construction surveying is to realise a construction at a site, not merely to make maps (Sec. 2.2). In some instances, however, a site map may be helpful in the design and construction process if it meets certain specific requirements (Sec. 2.3). Setting out a construction at a site implies an inversion of the mapping process, for which the same survey techniques are used (Sec. 2.4). The possibility for errors lurks in every phase of a survey; prevention and timely detection are at the basis of ‘good surveying practices’ (Sec. 2.5).
Chapter 3 begins with clarifying how in a surveying process real-world space is connected to some artificial mathematical space. Our real-world space is split up into two ‘flat’ spaces: horizontal vertical. In real space, two sorts of geometric quantities are measured: lengths between positions and angles between directions. These quantities must be geometrically related to the artificial mathematical space. And inversely, geometric quantities must be literally realised at a site before a construction can be built (Sec. 3.1). Points and lines have to be materialised, either temporarily or permanently, on a construction site (Sec. 3.2). Devices are used to measure lengths between positions along survey lines (Sec. 3.3). Square and non-square angles are used to realise directions both horizontally (Sec. 3.4 & 3.5) and vertically (Sec. 3.6 & 3.7).

Chapter 4 addresses accurate measurement of height differences (vertical lengths) over large horizontal lengths, which requires the use of a levelling instrument. Very limited space available for this chapter means that all that can be presented are the concepts of levelling (Sec. 4.1) and a description of common equipment (Sec. 4.2). There is no room to address applications. Methods are presented in a rather brief manner (Sec. 4.3), followed by tips regarding error prevention (Sec. 4.4).

Chapter 5, in two pages presents a summary of ‘good surveying practices’ as related to the methods and techniques presented in chapters 2, 3 & 4. It is the only part of the book demonstrating a clear cookbook style.

The glossary included after Chapter 5 comprises a description of most technical terms used throughout this Agrodok.

Advice for Further Reading is presented on the closing page.
2 Construction surveying comprises more than mere mapping

What construction surveying is about will be discussed in the first section of this chapter. The principal aim of construction surveying is to realise a construction at a site, not merely to make maps (Sec. 2.2). In some instances, however, a map of a construction-site can be helpful in the design and construction process if it meets certain specific requirements (Sec. 2.3). Setting out a construction at a site implies an inversion of the mapping process, for which the same survey techniques are used (Sec. 2.4). The possibility for errors lurks in every phase of a survey; prevention and timely detection are at the basis of all good surveying practices (Sec. 2.5).

2.1 What ‘construction surveying’ is about

‘To construct’ means ‘to build’ or ‘to put together’. The purpose of construction surveying is to carry out surveying measurements required for the realisation of a construction on a site. Such a construction might be a road, a school, a canal, a storage dam, or the like. What has to be ‘surveyed’, and how the required measurements must be accomplished, depends on both the construction and the site. A site cannot be described geometrically by means of a site map alone. A map describes a site geometrically, but in a horizontal sense only. In many cases, a site needs also to be described geometrically in a vertical sense. This is enabled via so-called ‘sections’.

Surveying procedures

When it comes to surveying activities within the framework of realising a construction, three steps must be clearly discerned:

- Firstly, preparing for the construction involves horizontal and vertical geometric description by means of surveying measurements. This procedure is generally called ‘mapping’ because the descrip-
tion is often provided by way of a site map. The initial result of a site survey, however, is a field sketch, not yet a site map.

- A site map has to be produced from the field sketch. A site map provides the basis for designing the layout of a construction with respect to the site. However, if dimensions are actually to be determined during and not in advance of construction, then the intermediate step of a site map is not needed.

- A construction layout is always to be set out on the site with its true dimensions and in the right position. The procedure called ‘setting out’ is also known as ‘staking out’. It requires the same measurement techniques as used for mapping a site.

**Construction methods**

Man-made constructions have been successfully built all over the world for millennia. Examples are bridges, irrigation systems, bench terraces, dams for water retention and all kinds of buildings. Though in a distant past surveying measurements were not required to build them, such structures still needed to be properly dimensioned. This could be achieved by following a construction method best described as ‘dimensioned during construction’. However, when a construction is to be realised according to some design, then ‘built as designed’ is the construction method to be followed.

Surveying measurements are required to properly set out the designed layout. The great pyramids in Egypt were constructed ‘as designed’ several millennia ago. Extinct Indian cultures in the Middle and South Americas realised impressive constructions also. By modern standards the surveying techniques applied here were very simple, despite the size and complexity of these historic constructions. Simplicity of surveying technology also characterised the construction techniques of the industrious Romans. A great number of their constructions still stand all over Southern Europe, the Middle East and North Africa.

**Planning a construction**

Designing a construction implies determining appropriate dimensions with the envisaged use of the construction in mind. Here are some examples:
An irrigation canal must be laid out with a specific slope and cross-section to enable water to flow with predetermined velocity and discharge.

A bridge must have a certain span and its construction must be strong enough to take the load of envisaged traffic.

A storage dam with an anticipated height must be strong enough to resist the pressure of the water behind it.

A school with a decided number of classrooms must provide space for an envisaged number of desks and seats.

**Drawing or sketching a planned construction**

A map implicitly reveals dimensions, because dimensions are contained in the graph itself. These are the signs on a map referring to real-world dimensions:

- a numerical scale ratio (for instance 1:500),
- a graphical scale bar (see figure 6),
- a grid of evenly spaced squares.

This is different from a construction drawing. In a construction drawing, all relevant measures and dimensions must be explicitly registered in numerical form.

The reason why a construction drawing explicitly and numerically provides all relevant dimensions is for reliability, because the drawing will be used in building the construction. Parts of the construction will be fabricated separately. When assembled on site, all parts must fit. Dimensional accuracy needs to be to a few centimetres, or even millimetres. It would be very difficult to achieve this accuracy if trying to derive dimensions of parts by calculation from measurements on a drawing made using an ordinary ruler. This would not only be inconvenient and cumbersome but, more importantly, unreliable. The use of a ruler is too crude. Deriving dimensions in this way is prone to errors, both in the measurements themselves and in the calculation of the true dimensions drawn from them.

Given the rural scope of this booklet, detailed construction drawings will be required very rarely. In most instances it will be sufficient to use a sketch illustrating all required dimensions of the planned construction. Of course, a check must be made to ensure that the envis-
aged construction fits the planned site. This is a matter of setting out cardinal dimensions on the site, as will be explained in Sec. 2.4.

**Drawing materials**

Ordinary office and drawing equipment can be successfully used for making maps and technical drawings. Sharp and hard pencils are a prime necessity. A simple ruler, preferably with a half-millimetre division, is needed for drawing straight lines and for measuring dimensions on the map. A pair of compasses can be used for drawing arcs with a specific radius.

Though blank white drawing paper is the best basis for a neat map, it’s more convenient to use gridded paper sheets. Office paper is available with grids of 1 centimetre and 5 millimetres. Even more convenient are special millimetre-grid sheets, available in several sizes. To measure or to set out angles on a map a protractor can be used with an angular division of one-degree, preferably half a degree.

### 2.2 Surveying a site

Making a map of a site is a difficult and cumbersome task subsequent to surveying activities on the site. It is important to decide whether the making of a true map is necessary. In many cases field notes and sketches made during the survey of the site will be sufficient to enable the setting out of a planned construction. Thus, *a clear distinction has to be made between ‘surveying a site’ and ‘making a site map’.*

**Recording data in a field book**

A site map has to be produced from data recorded in a field book whilst making surveying measurements on the site. The field book contains various types of data:

- **Sketches** to reveal all relevant terrain features in their proper relative positions, as well as points and lines used for the survey.
- **Tabulations** to record measurement data in an orderly manner.
- **Descriptions and annotations** easing the interpretation and use of measurement data.
- **Calculations** to check and assure correctness of measurement data.
The content of a site map cannot reveal any more information about the site than allowed by the data recorded in the field book, as illustrated by the two graphical representations in Fig. 1 and Fig. 2, which differ remarkably in their geometric characteristics.

![Figure 1: Part of an imaginary field sketch used to produce the site map shown in Fig. 2.](image)

**Need for and purpose of a site map**

Map making is a complex and difficult task. If map making cannot be dispensed of completely, it must be restricted to the required minimum. Such a ‘minimal map’ shows marked points and survey lines used in the measurements on the site, as well as relevant obstacles, if any. These points and lines are instrumental in the proper horizontal layout of the construction.

The map must show the required level of geometric detail. If, for example, a school must be built on a site, then the site map must allow for realising two objectives:

- designing the layout of the buildings correctly on the site map;
- transferring this layout from the map to the site.
Figure 2: Imaginary site map derived from the field sketch shown in Fig. 1. The initial scale of the original map was 1:100. The scale of the reproduction in the figure here is smaller, due to reduction in size. The length of the scale bar (appr. 23mm) enables establishing the actual scale: $23 \text{ [mm]} : 10 \text{ [m]} = 23 : 10,000 = \text{appr. 1:430}$.

The transfer task implies that the layout must be set out at the site ‘as designed’, which means in its proper place and with its proper dimensions. This is to be accomplished by way of surveying measurements (staking out).

**Representing height with symbols and sections**

The geometry of a map represents the geometry of a site in a horizontal sense, not in a vertical one. Information about height can be included on a map only by using of graphical symbols, not geometrically. Three types of symbols are in use; see Fig. 3:
A dot or other small symbol accompanied by a number. The number expresses the height in some unit of length, for instance metre. The point refers to the terrain position associated with that height. This is called a *spot height*.

A line accompanied by a number expressing the height. The line refers to all terrain points that feature that same height. This is called a *contour line*.

A *graphical symbol* to indicate an abrupt or steep transition in terrain relief, such as an embankment, an excavation, a dike or a scarp. This symbol is not accompanied by numerical information.

![Figure 3: Three graphical symbols to represent height in a map: (a) spot height; (b) contour line; (c) graphical symbol.](image)

We will illustrate this approach by means of two examples. The first example concerns the design of a road, the second preparing a site for irrigation.

When a road or canal, is to be built, information about height should be available in numerical form; graphical information is not sufficient for geometric design. In a case like this, the normal approach is to describe height by measuring sections, where such sections have been measured. A site map shows the locations of these sections by means of line symbols, as in Fig. 4. A so-called longitudinal section presents height information along the course of a road (or canal). Cross-sections perpendicular to this longitudinal section provide additional height information. Such cross-sections are crucial for determining the dimensions of a road (or canal) that has to be built or rebuilt. In Chapter 4 we explain how to accurately measure sections.
Figure 4: (a) Imaginary site map of a road crossing a streamlet via a dam with three culverts; dotted lines indicate one longitudinal section (SP1-SP4) and seven cross-sections (A-G). (b) Longitudinal section SP1 - SP4. For cross-sections A-G; see Fig. 5.

An area that has to be prepared for irrigation will be divided into plots requiring a surface with an even slope descending in a specific direction. Even when the natural surface is quite flat and smooth local corrections will still be needed because some places will be too high and others too low. By cutting the heights and filling the troughs, an evenly sloping surface can be realised. A pattern of well-distributed spot heights is required to enable the computation of a proper ‘cut-and-fill’ for each individual irrigation plot. A suitable measuring technique for the establishment of these spot heights is levelling, which will be discussed in Chapter 6. Designing an irrigation scheme, including the required ‘cut & fill’ per plot, is beyond the scope of this Agrodok.
2.3 Requirements for a site map

Map geometry
‘To map’ means to describe significant terrain features graphically by their shape, orientation and relative position, but at a reduced size. The resulting graph represents an imaginary horizontal plane on which terrain features are indicated by way of points, lines and other graphical symbols. Both a sketch and a map serve this description, but a site map differs from a field sketch in its geometric characteristics. True forms and dimensions cannot be derived from a field sketch without consulting the additional measurement data.

The main purpose of a site map is to use it to design on this map the layout of a construction to be built. To enable this, the relation between the true geometry of the site and the one represented on the site map must be known. A map that achieves this condition most conven-
iently shows true angles as well as a fixed ratio between any length at the site and the reduced representation of that length on the map. If the requirement of a fixed multiplication factor between true lengths and their reduced representations is satisfied, then the map is said to display a uniform scale, irrespective of position and direction. Mathematically speaking, this implies that a site map’s geometry has to conform the true horizontal geometry of the site. In this respect ‘to conform’ means: ‘showing any small area in its correct shape, at a reduced size’. A field sketch of a site does not and cannot obey this condition.

‘Scale’ in contrast to ‘scale number’
A map scale can be chosen freely in accordance with the required detail and use of the map. For instance, if the trajectory of a 0.2km-long irrigation canal has to be represented on an A4 map sheet, the graph is not allowed to exceed an area of 20 times 30 centimetres. Then the 0.2km trajectory must be reduced to less than 30cm to fit an A4 sheet, which implies a reduction factor of ‘0.2km to 30cm’. On a map, a so-called ‘scale bar’ incorporates the ratio between a true horizontal length on the site and the equivalent length on the map, see Fig. 6.

Figure 6: Scale bars: (a) expressing three different map scales, the smallest scale for A and the largest for C; (b) expressing the same scale for all three bars, which is the scale of A in (a).
A scale bar is especially convenient when the ratio is expressed in terms of, for instance, ‘yards to inches’. For accurate mapping, however, the reduction factor has to be used instead of the scale bar. An expression like ‘0.2km to 30cm’ is, however, rather impractical, because the ratio 0.2 to 30 results in a decimal fraction 0.006667 in which the dot represents the decimal point. Moreover, the ratio also includes ‘m to cm’, which means that the fraction cannot be applied without taking into consideration the length difference between the units, because 1 kilometre equals 1000 times 100cm = 100,000 centimetres (the comma in the number represents the separation between the ‘thousands’ and the ‘hundreds of thousands’).

The convention is to express the scale of a map with a whole and round dimensionless reduction factor. In the example, a dimensionless number can be achieved simply by replacing the unit ‘metre’ by ‘centimetre’. This equivalence in length is brought to the equation as well and results in 0.2x100,000cm to 30cm (i.e. 20,000 to 30). Now both lengths are expressed with the same unit (cm), which results in a dimensionless reduction factor of 667 (when rounded off to the nearest whole number).

A number like 667 is not very convenient for the conversion of true lengths at the site into corresponding lengths on the map, and vice versa. It is more practical to use a ‘round figure’ for the reduction factor, in this case 1,000. Hence 0.2km at the site is reduced to 20cm on the map. Thus, a factor 1,000 results in a stronger reduction than 667.

With respect to ‘map scale’, a clear distinction has to be made between the ‘scale’, which is a fraction with numerator ‘1’, and the ‘scale number’, which is the denominator in that fraction. This denominator is the reduction factor we introduced above. Hence, a map scale (S) is expressed as ‘1 divided by the reduction factor’, i.e. \( S = \frac{1}{s} \). Numerically a map scale is always a ratio much smaller than 1. A ratio 1 to 1,000 (also written as 1/1,000 or 1:1,000) is equal to the decimal fraction 0.0010, which is indeed 1.5 times smaller than 0.0015, which is the decimal equivalent of the fraction 1/667.
Appropriate map accuracy
Not all details of a site can or should be geometrically represented on a site map. When geometrically representing, for instance, a 1-metre thick tree trunk on a 1/10,000 scale map, this trunk has to be drawn with a circle as small as 0.1mm in diameter. This is technically impossible. Even at a twenty times larger scale (1/500) such a circle would still be no bigger than a tiny 2mm. Trained and skilled personnel will achieve a geometric mapping accuracy of 0.2mm at best using specialised equipment and drawing materials. Within the scope of this Agrodok it is not expected to meet such requirements. When ordinary office tools and drawing materials are used for map-making, a geometric mapping accuracy of about 1.0 mm is a more realistic figure.
At a map scale of 1/500, a 1-mm accuracy is equivalent to 500mm or 0.5m on the site. If construction details as small as 0.1m have to be retrieved with a map-accuracy no better than 1.0mm, the question arises as to whether the map scale has to be enlarged to 1/100. The answer is ‘generally not’, because the purpose of a site map is not to reveal construction details. A site map is primarily about the construction site, not about the details of the construction itself. Construction details should be documented separately and explicitly by means of a construction drawing, as we will discuss later. The purpose of a site map is to design and position the layout of a construction with a ‘required level of detail’ in respect of the existing features on the site. This will be illustrated with an example in the next paragraph. The example involves a new school to be positioned on a site amidst some houses and trees.

Appropriate map scales
Assume that the projected outline of the school is dimensioned at 25.3 metres long and 7.1 metres wide. These decimal figures indicate that geometric detail of the outline is relevant up to 0.1 metre or 100 millimetre. If the scale of the site map is chosen at 1/500, then the designed dimensions of the school need to be divided by 500 to get the equivalent dimensions for the layout to be put on the map.
Expressed in millimetres, the outline measures 25,300mm by 7,100mm. When reduced to the scale of the map, the outline becomes 25,300/500 [mm] to 7,100/500 [mm], which is equal to 50.6 [mm] to 14.2 [mm]. This result indicates that, for a geometrically true representation on the map, ‘better than 0.1mm’ mapping accuracy is required. This high accuracy is, however, not realistic in view of the 1mm accuracy that can be achieved using simple drafting materials. More important is that 0.1mm mapping accuracy is not required. This becomes obvious when considering the purpose of the site map. Assume that the site has been selected so as to make use of the shadow cast by some large trees on the area projected for the new school. The distance between a tree trunk and the building should be ‘at least a few metres’. This yardstick is not very precise but it clearly indicates that accuracy ‘better than one metre’ will be sufficient for an appropriate positioning of the layout of the school amidst the trees.

As regards an estimated mapping accuracy of 1mm, the reduction factor has to be ‘smaller than’ 1 [m] to 1 [mm]. As 1 metre is equivalent to 1,000 millimetre, this figure implies a reduction factor ‘smaller than 1,000’. This means a map scale of 1/1,000 or larger, so a 1/500 scale will be fairly good for the purpose at hand. A map scale of 1/500, however, enables representation of the site on the map with an accuracy of 0.5m. This value is equivalent to about five hundred times the required construction accuracy. The site map is thus definitely unsuited for a geometrical representation of construction details. As we stressed before, representing construction details is not the purpose of a site map. The representation of construction details clearly needs a drawing scale of 1/25 to 1/100.

**Summary of requirements for a site map**
Maps may serve many purposes, thus it is not feasible to lay down general requirements. Each map is made with a specific purpose in mind. Map characteristics, such as graphical symbols and scale, always reflect this purpose. The requirements listed below relate to site maps for use in simple construction surveying.
General
- Title, date and ‘author’ of the map; name and location of the site.
- Explanation of the purpose and content of the map in a ‘title block’.

Geometric aspects
- Scale and scale bar.
- North arrow, or some other reference for the orientation of the map.
- Marked survey points which indicate survey lines, benchmarks, etc.

Graphical symbols
- Clear graphical layout, enabling easy ‘readability’
- Consistent and uniform use of point, line and area symbols
- Conventional use of signs and colours

A good start for getting to grips with maps and map making is to look carefully at as many maps as one comes across.

2.4 Setting out a construction on a site

Setting out a designed construction on a projected site requires the same surveying techniques and tools as for mapping that site, but the objectives are different:
- **Mapping** refers to surveying measurements related to already existent features at the site.
- **Setting out** is about surveying measurements dealing with features that must be added to the site.

For instance, the corner points of a school building must be marked on the site by pegs or poles *at their intended physical positions* in respect of existent terrain objects like trees, large boulders, houses, a road etc. Moreover, the floors must be set out level and at the intended height *in respect of the terrain surface.*

Surveying measurements are needed to establish a geometric link between existent terrain features and the layout of the construction. As stated before in Sec. 2.1, there are two methods for realising a construction: ‘dimensioned during construction’ and ‘built as designed’.
In the former case, there is no need for a real design described by technical drawings. A site map is not required either. For setting out, a sketch with indications for the position and orientation of the planned construction on the site provides all information that is needed. We will explain this in Sec. 2.2. Measurements at the site are necessary, but these relate to the shape and dimensions of the construction only.

The situation is quite different and more complex when a construction planned on a site has to be ‘built as designed’. In this case, a geometric link must be established between the design drawing and the site before building can start. How this link is made depends on the geometric description of the site. Is it a field sketch or a site map? The geometry of a site map conforms to the site, whereas that of a field sketch does not, as the geometric differences between Fig. 1 and 2 show. Let’s have a look into this matter.

**Using a field sketch only**

The geometric link between sketch and site cannot be established by doing measurements with a ruler on the sketch because a scale is needed to calculate true lengths from lengths measured in the sketch. One solution is to make the sketch to scale, which implies that it has to be transformed into a site map. This solution will be discussed in the next paragraph.

On the other hand, there is no need for thorough mapping when it is obvious where the planned construction has to be positioned on the site, or when the site encompasses a flat and nearly empty space. In cases like these an appropriate position and orientation of the layout can be determined at the site. A line should be added in the design drawing that defines the baseline for staking-out the layout on the site. Two permanent point markers, to be positioned on the site, enable the transfer of this baseline from the construction drawing to the site. Starting from this transferred baseline, all other lengths and dimensions indicated in the design drawing can be set out by means of surveying techniques.
Using a prepared site map

When position and orientation cannot be freely chosen for the layout of a construction on the site, some kind of a site map is needed. Preliminary making of a site map is definitely the best approach for securing a reliable geometric link between the designed layout and the physical situation at the site. This approach comprises two steps.

![Figure 7: Imaginary and simplified examples of a design drawing (a) and a site map (b). Before the drawing (b) can be fitted into the geometry of the site map (a), the scale of the drawing needs to be made identical to the scale of the map; see Fig. 8.](image)

**Step 1: from design drawing to site map**

The first step in a setting-out procedure is to transfer the layout from the design drawing to the site map. Generally, the drawing and the map will have different scales, see Fig. 7. Consequently, the layout has first to be made to scale before it can be put into the map. Here is a simple procedure:

- Redraw the designed layout precisely on a separate piece of paper. The scale has to be equal to the scale of the site map.
Cut out the layout drawing along its edges.
Position the layout on the map within the projected space.
When the right position is found for the layout on the map, fix it to the map.

Suppose that the layout has to be positioned parallel to the map’s baseline; the result is that the map and the layout are now linked graphically, but not yet geometrically. Without such a correlation the layout cannot be set out on the site according to the design in Fig. 8.

**Step 2: from site map to dimensions that can be set out**
A geometric relation needs to be established between the designed construction and the site. Survey points are already marked at the site, as previously used when mapping it. These survey points are displayed on the site map (Fig. 2 and 8). The construction of the geometric relation between layout and site starts at the baseline over the survey points 1-2-3-4 and ends with a graph that shows true measures to be set out along survey lines at the site, see Fig. 9. First, survey lines needed for setting-out must be added to the site map. Thereafter, measures that need to be set out on the site must be noted along these survey lines.

**Adding survey lines to the site map:**
- When constructing on the site survey lines for setting-out the construction, the ‘point of beginning’ will be the existing survey point SP3. From SP3 starts the first survey line, which then runs along the initial baseline (SP1-SP4) and ends in point 41.
- Start with the site map, connecting the corner points of the layout (see Fig. 8) by means of four dotted lines perpendicular to the baseline. These lines are numbered 11-12, 21-22, 31-32 and 41-42.
- Indicate with dotted arcs the angles these lines have to the baseline, signifying that the angles must be set out square.
- Add parallel to the baseline the dotted lines 13-43, 14-44 and 25-35.
- Indicate the survey lines by full lines and mark the beginning of each line with an arrowhead.
- Add three diagonal survey lines needed for checking whether angles between survey lines are truly square.
Figure 8: Layout of the construction correctly scaled and positioned within the projected area on the site map. (Assume that the original map actually has a scale of 1:100.)
Figure 9: Graph showing the geometric link between the layout of Fig. 8 and the construction site. (The layout of the school is not indicated, to keep the graph simple.) The graph shows survey lines and measures needed to set out the designed layout at the site. The construction of survey lines and numerical assessment of measures are explained in the text.
Noting measures next to the survey lines

- First establish the position and orientation of the outline by means of the distance from SP3 to line 11-12 and the distance between the parallel lines SP3-SP2 and 13-43. Measure these two distances on the prepared map (Fig. 8) with a ruler and multiply each distance by the scale number of the map. If the map is at scale 1/100 and the measured distances are, for example, about 3mm and 6-7mm, then the required lengths to be set out are 0.30m and 0.65m.
- Then infer the measures to be set out along each survey line by means of the dimensions in the design drawing (Fig. 7). For instance, for the survey line from SP3 to 41 (Fig. 9), the first measure to be set-out is 0.30m. The next measure along this line is 5.18m, which is the result of $0.30 + 7.43 - 2.55$. Then follows 7.73m, resulting from $0.30 + 7.43$. The survey line ends in point 41 at a distance of $16.28 + 0.30 = 16.58m$ from SP3.
- Finally, establish the lengths of the three controlling diagonal survey lines by applying the Pythagorean theorem. For instance, the length of the diagonal that controls the square angle between the lines 11-14 and 11-12 is 6.64m. This value results from taking the square root of $[4.50^2 + (5.18 - 0.30)^2] = 44.06$.

2.5 Dealing with errors and mistakes

Surveying measurements are notoriously error-prone and therefore the occurrence of inaccuracies has to be dealt with. Preventing and countering errors is one of the most important aspects of surveying. But not all inaccuracies can be considered errors. To cope with errors effectively one needs some understanding of the occurrence of inaccuracies and their potential causes and consequences.

An important distinction has to be made between two kinds of inaccuracies that impair survey measurements:

- **Real errors**, which are called ‘blunders’. These inaccuracies are the result of unintentional human mistakes. As complete prevention of such mistakes is impossible, they should be detected and, if possible, neutralised.
Most abnormal inaccuracies are caused by unintentional mistakes. Surveyors call these ‘blunders’. Blunder detection is at the heart of all survey work.

- **Normal deviations in the measurements.** This kind of inaccuracy is inherent to the limited precision of any measurement technique. Ideally, this should be the sole source of inaccuracy. Improving the precision of measurements requires the use of more sophisticated surveying techniques.

In general: the more simple the surveying equipment, the less precise will be the resulting measurements.

**Some causes of unintentional blunders**
The human capacity for making mistakes is unimaginable! Therefore it is impossible to list all the errors that are likely to occur in construction surveying. Fortunately, the causes of errors can be catalogued according to a quite limited number of types.

**Malfunctioning of equipment**
It is rarely evident from external observation whether or not a surveying device is functioning properly. A malfunctioning device unintentionally results in erroneous measurement data. In general, the simpler the equipment, the more resistant it will be to malfunction; but even the simplest device needs care and maintenance. Regular checking of equipment belongs to what is called ‘good surveying practice’.

**Misuse of equipment**
In the hands of a non-professional, even the most precise and best-maintained piece of equipment is unlikely to produce accurate results. Simple surveying equipment needs to be used properly – often a complex task, as will be clarified in Chapters 4 and 5. The more complex the equipment *in its use*, the more likely will be the occurrence of handling errors. This does not imply that technically complex equipment is difficult to use, as the opposite is true. Modern computerised surveying instruments are both very precise and very easy to use. Un-
Fortunately, due to the technical complexity and high price tag, such equipment is beyond the scope of this Agrodok.

**Incorrect registration of data**
Most notorious is a slip of the pen when writing down the output of a measuring-instrument. Tracing such an error during fieldwork is very difficult. One special category of registration errors is omission: the result of a necessary measurement is not included in the field notes. The cause of registration errors is either inadequate remit for the field notes, or lack of professional focus during fieldwork.
Survey measurements should be set up in such a way that false data can be detected and subsequently discarded without a need for repeating any measurement. Moreover, the set-up should ease timely detection of omissions.

**Incorrect calculations**
Slips of the pen may also occur frequently during the processing of measurement data, both in a graphical sense (map) and in a computational one (geometry). Most calculation errors are, in effect, errors in notation. An important difference between these and errors in the registration of measurement data lies in the control. A calculation can be checked and corrected if necessary, but wrong registration of a measurement (if detected!) cannot be corrected. However, wrong data can be discarded if measurement data show sufficient redundancy.

**Good surveying practice, the number-one priority**
A successful strategy for counteracting surveying errors is not so much different from good accountancy practice. Frequently used terms like ‘reliability’, ‘precision’ and ‘accuracy’ refer to basic surveying concepts. The next paragraph explores these terms and finally conducts us towards the important concept of ‘redundancy’.

**Accuracy consists of ‘precision’ and ‘reliability’**
The result of a measurement needs to be accurate. Accuracy, however, has two aspects. One aspect is ‘precision’, which relates to differences in results (dispersions) that occur when, for instance, a length meas-
Construction surveying comprises more than mere mapping. The other aspect of accuracy is 'reliability', which indicates whether the result of a series of measurements is true and can thus be trusted. The meaning of the terms precision and reliability can be conveniently illustrated with the results of an imaginary rifle test.

![Image of rifle test results](image_url)

**Figure 10**: The concepts of 'precision' and 'reliability' can be illustrated with the results of a shooting test, which is performed to check and adjust the sight of an imprecise, normal rifle and that of a precise rifle belonging to a sharp-shooter. The dispersal of a round of ten hits over the target illustrates the precision of a rifle; their average position (indicated by a cross) determines the reliability of its sight.

It is impossible to know whether a rifle’s sight is upset unless a shooting test is performed on a target like the one in Fig. 10. The figure shows four patterns of ten hits each. In our imaginary rifle test an experienced gunman has fired all shots, using two different rifles: a normal rifle and a sharpshooter rifle. The patterns in Fig. 10 indicate that the hits of the sharpshooter rifle are less dispersed than those of the normal rifle. These results do not, however, guarantee that using a
sharpshooter rifle will always result in more bull’s-eyes. An upset sight does not decrease the precision; rather it makes targeting unreliable. In Fig. 10, out of ten shots only one is a bull’s-eye, against three for a normal rifle with the same aberration of its sight. With both sights properly adjusted, the number of bull’s-eyes for the sharpshooter rifle improves to nine, whereas for the normal rifle the result remains three. The average positions of all ten hits per rifle show that the two rifles are equally reliable. There is no longer any aberration, as for both rifles the average of all ten hits (marked by a cross) is now in the centre of the target.

This result has a parallel in surveying practice: the use of a precise instrument does not necessarily produce the expected ‘true’ result, due to systematic error. Such an error may remain undetected. The average result of repeated measurements cannot disclose a hidden systematic error if that error has the same influence on each individual measurement. Such an error can only be detected via a set of independent reference measurements, or by using a benchmark. In the shooting experiment, the target provides a same reference for each single shot. In a survey, however, the inclusion of a benchmark for every individual measurement is highly impractical.

The detection of errors in survey results is based on applied geometry. This means the application of mathematical rules, offering opportunities to detect systematic errors and other blunders in a set of measurement data. The trick is to include more geometric elements in the design of the survey than the number of elements minimally required for geometrically solving the problem. Such a solution is then said to be geometrically ‘over-determined’, which means that some of the elements are ‘redundant’. In this context, ‘redundant’ definitely does not mean ‘superfluous’, ‘needless’ or ‘unwanted’; it implies the opposite.

**Blunder detection using adequate redundancy**

The benefit of a redundant geometric solution can be illustrated by means of an example about the replacement of a missing door (Fig. 11a). In this example, the shape and size of a new door must be assessed via measurements revealing size and shape of the remaining doorframe. A carpenter should slightly reduce the measured dimen-
sions in order to prevent the door from jamming in the frame. Therefore, measurement requires the use of a measuring tape with millimetre divisions.

Figure 11: Geometric reliability explained graphically. (a) An existing skew doorframe needs a new door. (b) Measures (in mm) of all four sides alone cannot reveal the correct shape. Addition of one measured diagonal determines the shape. (c) However, a blunder made in the dimension of the upper side (minus 27mm) cannot be detected. (d) The error does not appear before the door is placed in its frame.

Conclusion: There is a complete absence of redundancy in the geometric construction. There is no reliability at all.

Assessing no more than height and width of the doorframe suffices, but only if the doorframe is supposed to be exactly rectangular. The carpenter mistrusts this supposition. Therefore he checks whether the two vertical posts show the same lengths. He also checks whether the lengths of the two horizontal parts are equal. His measurements demonstrate that the lengths of the opposite sides are indeed different. The problem is that the lengths of the four sides alone cannot yet reveal the
actual shape of the doorframe. Our carpenter solves this problem by additionally determining the length of one diagonal. With the diagonal serving as a baseline for the geometric construction of the new door, he can precisely derive its shape and size from the measured lengths of the doorframe (Fig. 11b). But ... how reliable will be the resulting shape and size? The carpenter first checks whether all four sides of the doorframe are straight. Let us suppose that this is true. Suppose next that the carpenter has correctly measured the width both along the upper side (796 mm) and along the bottom (772 mm). Now suppose that for the length of the upper side he unintentionally writes 769 instead of 796 without noticing the writing error. Though he repeats his measurements, he overlooks the slip of his pen. (Many people easily overlook a reversal in the position of two successive digits in a number.)

![Figure 12: Geometric reliability explained graphically (cont.).](image)

(a) Addition of the second diagonal measure reveals (b) the occurrence of an error, but (c) the error cannot be located.

Conclusion: Redundancy is too weak; consequently, reliability is insufficient.
The carpenter starts constructing the door without being aware of the blunder in the length of the upper side of the doorframe. The available measurements do not allow for any redundancy in the geometric construction of the door. Therefore, when he tries to mount the finished door in its frame (Fig. 11d), the carpenter discovers too late that the upper side of the door is too small. What could he have done to detect his blunder much earlier whilst still constructing the door?

If he had measured the second diagonal too as a supernumerary measurement, see Fig. 12a, then the construction would have shown geometric redundancy. In this case, by using all six dimensions for determining the size and shape of the door in his workshop, the carpenter discovers the six dimensions to be contradictory: they do not fit. The misfit warns him that an error has stolen into the noted dimensions somewhere. However, the ‘weakness’ of the redundancy (one single supernumerary dimension) prevents him to assess which of the dimensions is false. It is always the last added dimension that fails to fit, see Fig. 12b & c. He is forced to go back to the doorframe to check all six dimensions. This would not have been necessary if our carpenter had been less economical in his measurements, as the next solution illustrates.

The following solution does not require diagonals. The carpenter starts with fixing two straight laths to the doorframe. One lath is set horizontally with a level; the other lath is set vertically using a plummet. These two laths provide a reference frame of two perpendicular straight lines that divide each side of the doorframe in two parts (Fig. 13a). In addition to the lengths of the sides of the doorframe, the carpenter measures:

- the lengths of the two laths (760 and 1,963), see Fig. 13a.
- the lengths of all eight parts of the doorframe, as well as of the four parts of the two laths, see Fig. 13b.

These sixteen measurements together enable construction of the door with sufficient redundancy to detect and correct the blunder by means of two checks.

The first check is numerical and can be done on the site. For each two parts, the summed lengths must equal the corresponding total length (which has been measured independently). Even though a measured
total length may show a slight difference (of a few millimetres) from the sum of the two corresponding parts, this check warns that a blunder must have been made in one of the three measurements of the upper side (Fig. 13c). However, the false measurement itself cannot be detected.

**Figure 13: Geometric reliability explained graphically (conclusion).**
(a) Two perpendicular lines replace diagonals. (b) Auxiliary measurements improve geometric control. (c) Each measurement can be checked numerically via two other independent measurements. The last row indicates a blunder in one of the three measurements for the upper side: the length must be either 798 or 769. (d) When checking the positions of the two upper corners graphically by means of the lengths in (b), it becomes apparent that 769 is too short and must thus be discarded.

Conclusion: Sufficient redundancy results in adequate reliability.

Note that the carpenter correctly measured the upper side as 796, but that he subsequently noted the length as 769. If he had performed the sum check on the site, he would have detected a contradiction in the three measurements for the upper side, because 250 + 584 = 798, which differs too much from the noted 769. Let us assume that he is in a hurry and forgets to do this check. As in the previous case, he will
detect the sum error only later, when constructing the door in his workshop.
Back here, the redundant measurements allow him to do a second check that enables him to detect the corrupted length. He starts with constructing graphically the crossing of the two perpendicular lines. Then he constructs the four corner points from the endpoints of the perpendicular lines using the parts of the sides: the lengths are shown in Fig 13b. Each resulting side comprises three points: one endpoint of the cross and two corner points. As a side must be straight, these three points must be on a straight line. Fig. 13d illustrates that if the right-hand side of the door is to be straight, then the length of 769mm noted for the upper side is too short, whereas 798mm is (approximately) correct. Thus, the measure 769mm (Fig. 13a) must be wrong and has to be discarded.

Conclusion: stick to ‘good surveying practices’
The analogy between this carpenter’s problem and a surveying problem becomes apparent when we replace the doorframe by a map of a building site, and the door by the designed layout of a building to be set out at this site. The elimination of an error without the need to do additional measurements is exactly the role geometric redundancy plays in survey work.
Simple construction surveying should be no exception to this rule. Prevention of blunders starts with a well-documented field sketch of the site. This sketch should unambiguously reveal all relevant terrain features and the measurements made. Along with the related documents, it forms a geometric basis for the design and construction process. Therefore always check during fieldwork whether measurements allow enough geometric redundancy to enable the detection and correction of blunders.
Always register all relevant data in an orderly fashion in a well-documented record, preferably using purpose-made forms. Indicate all changes in this record in such a way that the original data remain available and readable. Make ample use of explanatory notes with unambiguous references to registered measurement data. When feasible, add photographs of the site.
3 Surveying methods and techniques

A surveying process connects our real space with an artificial, mathematical space. In real space two sorts of geometric quantities are measured: lengths between positions, and angles between directions. These measured quantities must be correlated geometrically in the mathematical space. Inversely, geometric quantities must be literally realised at a site before a construction can be built (Sec. 3.1). Hence points and lines on a construction site need to be materialised, either temporarily or permanently (Sec. 3.2). Devices are needed for measuring lengths between positions along survey lines (Sec. 3.3). Accurate measurement of height differences (vertical lengths) over large horizontal distances requires the application of a levelling instrument. (This subject will be discussed separately in Chapter 4.) Square and non-square angles are used to determine, or to set out, horizontal directions (Sec. 3.4 & 3.5) and vertical directions (Sec. 3.6 & 3.7).

See Chapter 5 for a summary of ‘good surveying practices’.

3.1 Realising lengths and angles in two planes

Space is to be described in a relative sense

From real space to abstract planes
In surveying, the physical space in which we live (‘reality’) is described in an abstract way by means of two artificial mathematical planes, i.e. such planes do not exist in our physical space. One of these planes is ‘vertical’, i.e. parallel to the direction of a plumb-line. The other plane is perpendicular to the plumb line and therefore designated ‘horizontal’. The vertical direction is used to describe ‘height’, whereas the horizontal plane serves for describing ‘positions’. For both descriptions, the same geometric concepts and conventions apply.
From position and direction to length and angle

On a mathematical plane space is described geometrically by two quantities: positions (‘here’ or ‘there’) and directions (‘from here to there’). A position, however, can only be described relative to another position, i.e. by a displacement. This description requires both length and direction of the displacement from the one position to the other.

Each two positions define a direction. Though a direction is subsequently determined in itself, directions must also be expressed in a relative sense when more than two positions are involved. (That is, unless positions are described in an absolute sense, which is beyond the scope of this Agrodok.) A rotation from one direction to another is called an angle; see Fig. 14.

![Diagram](image)

Figure 14: Three positions expressed relative to each other via displacement and rotation: (a) from ‘here’ to ‘there’ and back to ‘here’ again, (b) from ‘here’ to ‘there’ and to ‘over there’.

Units of physical length

Each relative position, i.e. displacement (length) from one position to another, has a certain dimension. Lengths need to be made ‘comparable’ by expressing their dimensions in a ‘unit of physical length’. This unit is ‘built-in’ in measuring devices such as a ruler, for office work, and a surveying tape for fieldwork.

Two widely used systems for units of length are the metre-centimetre-millimetre system and the yard-foot-inch system. These systems are supposed to be self-explanatory. Only the metre-centimetre-millimetre system is used in this Agrodok.
Angular units

All angles need to be expressed using the same angular unit. The most common way is to define a unit of angle by dividing a circle into four equal parts and subdividing each quarter into ninety equal parts, called degrees, and noted 90°. Hence a full circle comprises 360°. A decimal number like 34.23 can express subdivisions of a degree. Another way to express subdivisions of a degree is by using ‘minutes’ and ‘seconds’ in the same way as for the division of one hour of time. In simple construction surveying the measurement of angles shows a precision that is at best several decimal tenths of a degree, or several tens of minutes.

In professional surveying the ‘grad’ or ‘gon’ is a frequently used angular unit. This unit is based on a quarter circle divided into one hundred equal parts, noted 100g. Thus 90° and 100g both denote a square angle. Subdivisions of 1g are expressed as a decimal fraction only.

The decimal 90° angular system is used throughout this Agrodok.

Clockwise and anti-clockwise angles

There are always two ways to define the direction of a rotation: clockwise and anti-clockwise. If, for instance, a clockwise rotation is defined as ‘positive’, then the anti-clockwise rotation is ‘negative’. This is rather confusing and a source of many computational errors. In surveying and navigation the clockwise direction is ‘positive’, whereas in mathematics the anti-clockwise direction is positive. The numerical difference between a ‘positive’ and a ‘negative’ angle of the same size is illustrated for a clockwise defined angular system in Fig. 15a and b. In Fig. 15a, A→B is the ‘zero’ reference (‘from’) direction, whereas in Fig. 15b this is A→C. In Fig. 15a, the clockwise angle from A→B to A→C has a positive value (+34°) because A→C is at the ‘right-hand side’ of the reference direction A→B. In Fig. 15b, however, the anti-clockwise differential angle from A→C to A→B is ‘negative’ because A→B is left of the reference direction A→C, and thus its value (–34°) is ‘smaller than zero’. The differential angle from A→C to A→B can, however, also be expressed in a clockwise (‘positive’) direction, as shown in Fig. 15c. The numerical sum of the ‘positive’ angle and the ‘negative’ angle must be exactly a full circle. The
value of the ‘positive’ angle therefore follows from the addition of a full clockwise circle (+360°) to the ‘negative’ angle (–34°). The numerical result is a clockwise (thus ‘positive’) angle of 326°.

Figure 15: The direction chosen as the reference direction determines the angle of rotation. The rotational angles in (b) and (c) are numerically different. Despite the difference, both situations are geometrical identical, as the addition of a full (360°) circle proves: (–)34° + 360° = 360° – 34° = 326°.

Horizontal and vertical angles
On a horizontal plane, the division of a full circle into four equal parts provides the four cardinal horizontal directions of the compass card: North, East, South and West, see Fig. 16a. North serves as the reference direction for expressing compass angles up to 360°: ‘azimuth angles’. Clockwise azimuth angles are defined as ‘positive’.

On a vertical plane, the division of a half circle into two equal parts provides three cardinal directions orientated in respect of the direction of the plumb-line, as illustrated in Fig. 16b:
- Nadir (direction of the plumb-line),
- Horizon (perpendicular to the plumb-line),
- Zenith (opposite to direction of plumb-line).

Angles are expressed in two different ways. One is by zenith angles, from Zenith (0° to 180°; Fig. 16b); the other is by vertical angles, from the Horizon (Fig. 16c). In the latter case, ‘positive’ angles above the Horizon indicate ‘elevations’ (to +90°). Angles below the Horizon express depressions, assigned ‘negative’ (down to –90°).
Determining positions using geometric elements

Complex geometric constructions
Geometric constructions based on connected triangles and polygons provide the basic concepts of surveying. As the use of these concepts is actually a matter of applied mathematics, they are far beyond the scope of this Agrodok.

Polygons are geometric figures that show more than three sides; these can be either closed, like a triangle, or open. Closed polygons begin and end at the same point (i.e. position), whereas open polygons begin and end (close) in two different points. Surveying by measuring open polygons is called ‘traversing’.

Closed polygons with one or more square angles play an important role in surveying. A rectangle is a quadrangle of which all four angles are square. The geometric characteristics of a rectangle bring an additional geometric condition: opposing sides (lengths) must have identical dimensions. This condition provides a simple means of checking survey measurements. A rectangle with an additional diagonal makes two rectangular triangles.

Figure 16: Expressing angles: (a) Horizontally, by ‘azimuth’ angles from North. (b) Vertically, either by ‘zenith’ angles from the apex of the sky, or (c) by angles above (‘elevation’) or below (‘depression’) the Horizon.
Rectangular triangles provide a very simple means for the creation of perpendicular lines. These perpendicular lines lie at the heart of simple construction surveying. Therefore we will explain and apply some basic characteristics of triangles in subsequent sections of this chapter.

**Assigning a baseline**

Each direction must be expressed in reference to another direction, which results in a ‘from-to’ angle, as is illustrated in Fig. 15 & 16. Before determining any angle, one direction must be defined and set out (marked) beforehand. In construction surveying horizontal directions are rarely determined relative to North, as to be discussed in Sec. 3.5. Consequently, the direction must be set at an arbitrary value. This direction provides a so-called ‘baseline’ and its direction is the reference direction. The baseline serves as the backbone of the entire geometric construction resulting from a survey.

**Triangular constructions**

![Diagram](image)

*Figure 17: Closing a triangle by returning to ‘here’ via two other positions. (a) The three displacements (lengths) and two of the three angles of rotation form a closed triangle. (b) The sum of the three angles of rotation must form a full circle.*

The simplest surveying problem involves establishing relative positions of three points by constructing a triangle, as shown in Fig. 17a. Starting at any one of the three key points, return to that position is effected by successively constructing all three sides of the triangle with their proper lengths and directions. In the example shown, all
angles are anti-clockwise. However, if the direction of rotation is indicated (by ‘+’ and ‘−’ signs), then clockwise and anti-clockwise angles can be used indifferently. The sum of the three angles of rotation (all expressed in the same direction) precisely equals a full circle, see Fig. 17b.

The Pythagorean theorem (3-4-5 rule)
Square (90°) angles are widely used both in construction surveying and in the constructions themselves, because square angles can be constructed or established quite simply. How to construct them on a site, we will explain in Sec. 3.4 (horizontal plane) and Sec. 3.6 (vertical plane).

![Figure 18: (a) Triangles with a 3:4:5 side-ratio exactly fulfil the Pythagorean theorem. (b) Some examples of near-square triangles that do not exactly fulfil this theorem.](image)

One method is based on the unique geometric characteristic of square triangles expressed by the Pythagorean theorem. This states that a triangle is square if the squared dimension of the longest side numerically equals the sum of the squared dimensions of the two other sides. A popular designation of this theorem is the 3-4-5 rule illustrated in Fig. 18:

\[(5 \times 5) = 25 \quad \text{and} \quad (3 \times 3) + (4 \times 4) = 9 + 16 = 25\]
Every triangle with sides that show a ratio of 3 : 4 : 5 will exactly fulfill the Pythagorean theorem. A near-rectangular triangle with a side ratio of 3.000 : 3.000 : 4.243 is not precisely square. Neither is a triangle with the ratio 3.000 : 3.000 : 4.242, because:

\[(3 \times 3) + (3 \times 3) = 18\] whereas \[(4.242 \times 4.242) = 17.994\]

But this difference between 18.000 and 17.994 is not relevant, as we will explain below. From a surveying point of view, both triangles in Fig. 18b are rectangular. Thus, we must make a practical distinction between mathematics and surveying

**Meaningful units and significant decimals**

A square triangle must be set out in the real physical space of a construction site by means of physical units of length, see Sec. 3.4. A triangle with sides of 3,000mm, 3,000mm and 4,242mm, see Fig. 18b, is mathematically non-square. But is it also non-square in practice too? In construction surveying accurately measured lengths have a precision of 1-2 centimetre at best. Hence, a triangle with sides of 3.00m, 3.00m and 4.24m is practically square. And a triangle of which the sides measure 3 metre, 4 metre and 5 metre, respectively, will only be square in practice if set out with ‘better-than-one-centimetre’ precision. Moreover, sides indicated as 3.000m, 3.000m and 4.244m suggest that they have been measured with ‘one-millimetre’ precision. Such high precision, however, is not realistic for simple construction surveying.

### 3.2 Materialising geometric elements

**Using point markers**

Some survey points have to serve as a reference; for instance, two points that define a baseline. A point for horizontal reference is called a ‘control point’. A point that serves for vertical reference is called a ‘benchmark’. Some points are used for both types of reference. For
easy retrieval, the location of a reference point needs to be marked clearly, precisely and permanently by means of a ‘monument’.

Figure 19: Permanent point markers must be durable.

There are numerous ways to realise a durable, stable and accessible monument. One easy way is to chisel a cross in an outcrop of solid rock. When no solid rock is available, a monument can be made from various durable materials, see Fig. 19, such as a sufficiently long nail in road-surfacing material, an iron bar in weathered rock, an iron pipe in firm soil or a pipe cast in concrete (soft soil). Materials for temporarily marking survey points are, for instance, a lath, a peg or a hub (Fig. 20), a chaining pin (Fig. 21) or a range pole (Fig. 22).

Purpose: To mark a location or a height.
Material: Wooden lath; length 20 to 50cm; cross-section 2 × 3cm to 5 × 5cm; pointed or not pointed, depending on purpose and soil characteristics.
Use: Must be supplied with point identification. Place with a wooden hammer. Keep them vertical!

Figure 20: Wooden point markers.
Figure 21: Chaining pins held together with a ring.

Purpose: To indicate points that have been set out or that are to be used during chain (tape) measurements.
Material: Galvanised 5-6mm iron-wire about 30cm long; one end is bent into an eye (yellow or red painted), the other end is pointed.
Use: In combination with a tape of 10-50 metres, see Sec. 3.3. Ten yellow marked pins and a red one are kept together by a ring that can be opened. The eleven pins together span ten times a chain length, see for explanation Sec. 3.3.

Figure 22: Setting a range pole vertically (a) with a plumb bob (b) or staff level (c).

Purpose: To mark points along a survey line, or to indicate a line of sight to a survey point.
Material: Mostly wood with a sharp iron tip; length 1.5 to 2.5 metre; round with a 3 to 4cm diameter; painted red & white in 25cm sections to improve visibility. Bamboo can also be used. For precise work, a range pole must be dead straight.
Use: To be placed by pushing the pole firmly into the ground (soft soil only) or by using a self-made supportive tripod (three wooden sticks and a rope). For precise positioning (points or lines) a range pole must be set vertically by means of a plumb bob or a staff level (circular level, or bull’s-eye level).
Figure 23: Aligning points in a horizontal sense: (a) extension; (b) intermediate.

Figure 24: Line of sight cannot be completely overseen (a) across an elevation, or (b) over a depression. Intermediate points set out (c) using an iterative procedure (view from above).

Step 1: Select two points (C and D) from where the range poles at A and B are both visible; set range poles at C and D approximately aligned with A-B.

Step 2: View from C to B; align D as illustrated in Fig. 23b (view 1).

Step 3: Then view from D to A; align C in the same way (view 2).

Step 4: Repeat the cycle of two alignments (view 3 & 4); proceed until C-D-B is aligned with D-C-A.
Setting out lines of sight
The foundation of a construction must be set out correctly not only in a horizontal sense but also in a vertical sense at the same time. Hence construction lines must be set out and aligned both horizontally and vertically.

Range poles for horizontal lines of sight
If set properly vertical, range poles enable the alignment of points in a horizontal sense, see Fig. 22. For a line that is only tens of metres long it is sufficient to place one range pole at one end and a second at the other. Setting and aligning additional poles can extend a short line. In the same way, intermediate points can be set out, see Fig. 23. When setting out a survey line over uneven terrain, not all points of a line will be visible from either end of it. In such cases, intermediate points can be aligned using the iterative procedure illustrated in Fig. 24.

Figure 25: Boning rods: (a) starting rod or intermediate rod; (b) end rod.

Boning rods and batter-boards for vertical lines of sight
The vertical alignment of points along a line requires the use of horizontal sight rods (boning rods), see Fig. 25 & 26. Batter-boards (Fig. 27 & 28) are used to create construction lines vertically above pegs that mark a (horizontal) outline, see Fig. 29. How batter-boards can be used for the construction of a brick foundation is illustrated in Fig. 30. Batter-boards can be made of the nearest thing available.
Figure 26: Aligning an intermediate peg in a vertical sense by means of boning rods.

Step 1: Set pegs at both ends of a line with their heads at the elevation(s) required. Set intermediate pegs aligned (Fig. 23b) and spaced as required (Sec. 3.4, Fig. 33).

Step 2: Position the starting rod and the end rod (by a person at each end). Position the intermediate rod on one of the additional pegs (third person). The person at the starting rod aims his line of sight just over his own cross-bar to dividing line between black and white on the end rod. The person at peg C gets instruction to hammer it deeper into the ground.

Step 3: Repeat steps 2 & 3 until the height of the intermediate rod at C is aligned with the line of sight.

Figure 27: Construction of a simple batter-board (for soft ground only).

Drive two large pegs of sufficient length firmly into the ground at appropriate positions. The horizontal cross-bar is nailed to these pegs at the required level, which is 0.33m above the benchmark height, see Fig. 30 for explanation.
The level at which the crossbars of the batter-boards must be set is determined in reference to the height of a nearby benchmark. Establishing the height of a benchmark requires a technique called ‘levelling’. We explain levelling techniques in Sec. 3.6 and in Chapter 4.

Figure 28: Construction of a re-usable batter-board (for firm ground or stone floor).

Figure 29: Fitting a string over two batter-boards to mark a construction line.
Step 1: Reference height.
Set one peg with its head at a specified level. This benchmark height is used as a reference to set the cross-bars of the batter-boards at the level depicted in the design as the foundation.

Step 2: Construction lines.
Put eight batter-boards in place and fix four string lines perpendicular over the pegs. To ensure that the four string lines will be in the same horizontal plane, all four cross-bars must be set at exactly the same level. In the example, the cross-bars must be placed 33cm higher than the head of the reference peg. Use a levelling instrument to establish for each batter-board the height of one of its peg-heads in respect to the reference height, see Chapter 4. The difference in height must be bigger than the required 33cm, for example 41cm. Then use a carpenter’s rule to indicate the correct height on the batter-board’s peg at 41-33cm (8cm) below the head.

Step 3: Brickwork.
Build up the brickwork foundation to the level of the string lines. Regularly check plumb-lines at corners.

Figure 30: Using batter-boards and string lines to set out and lay a brick foundation.
3.3 Measuring length along a line (‘chaining’)

**Devices for measuring length**

The surveying term ‘chaining’ refers to measuring lengths by means of a survey chain or tape measure. A real chain (Fig. 31) is robust, but due to its weight is not convenient in use. More importantly, a chain lacks division into decimetres and centimetres.

![Survey chain: durable, but heavy and missing gradations.](image)

- **Purpose:** As accurate subdivisions are lacking, primarily suitable for measuring a (multiple of) full length(s).
- **Material:** Tempered steel wire links, about 25cm each, total length 10 or 20m; an adjustable brass-handle at each end to calibrate length.
- **Use:** To be used with survey pins for indicating begin and end points when chaining spans of several full lengths.

*Figure 31: Survey chain: durable, but heavy and missing gradations.*

For length measurement the usual tool is a steel tape on a reel, see Fig. 32. Lengths can also be measured with an optical surveying instrument. We will present this optical technique in the next chapter, as it is indispensable when using a levelling instrument.

Do not use a tape measure made out of fabric or plastic material. These materials are not durable and will suffer stretch under hot conditions. Tapes made of plastic or fabric are meant for indoor use only. **Always use a steel tape.**

**Using a steel tape (‘chaining’)**

Steel tapes most commonly show centimetre subdivision. Millimetre division, as on a carpenter’s ruler, is useless for normal survey work.
The gradations marked on a tape, do not always start at the beginning of the tape, see Fig. 32c.

Figure 32: (a) Steel tape, normally 30m long. (b) Gradations and lettering may differ. (c) Gradation may begin with the 'zero' at an odd position!

Figure 33: Chaining measurement 'from' peg A 'to' peg B. The 'Rear Chain' holds 'zero at peg', the Head Chain reads distance read at peg (4.52 metre) and then notes the reading.
Chaining requires two persons working together and communicating. The person holding the ‘rear chain’ keeps the ‘zero’ at the correct position. The person holding the ‘head chain’ reads the tape and keeps notes of the measurements, see Fig. 33.

Good chaining practices

- **Maintain good alignment**, both horizontally and vertically (proper tension), because the shortest distance between two points is a straight line.
- **Measure horizontally**, because surveyed lengths are always meant horizontally. If horizontal measurements are not practical on a slope exceeding a few degrees, a length measured along the slope needs correction (‘reduction’) to its equivalent horizontal length. This length reduction is explained in the next paragraph.
- **Measure all lengths twice** (back and forth) in order to detect mistakes, see Sec. 2.5.
- **Communicate loudly**; see the step-by-step procedure below.

Measuring lengths exceeding one tape length: a 5-step procedure

**Step 1.** The Rear Chain (RC) holds the end of the chain at the beginning point of the length, at the mark, for instance a peg or range pole. He gives direction to the Head Chain (HC), who walks in the direction of the end point. If necessary, the HC must clear the chaining path. The tape comes off the reel whilst he is walking. When the HC reaches the end of the tape he looks back to the RC. The RC instructs the HC to go left or right in order to get in line with the mark (range pole) at the end point of the line that is currently measured.

**Step 2.** Now the first full tape length must be marked by means of a chaining pin. The RC holds the zero mark of the tape to the mark on the ground and resists the tension applied by the HC. He yells, “mark”. The HC now pushes a chaining pin into the ground at the end mark of the tape. Then he releases tension and yells “next”. The HC moves ahead, dragging the tape behind him. The RC follows the end of the chain and yells “chain” when it is near the pin the HC just set.

**Step 3.** The cycle of steps 1 & 2 is repeated until the HC reaches the end mark of the line. He then reels up the tape until the RC yells
“chain”. The RC holds the zero mark of the tape at the last pin. The HC applies tension. When the RC yells “mark” he reads the tape where it is over the mark on the ground at the end of the line, see Fig. 33. He yells “OK” to the RC, releases the tension and notes the reading; for instance 4.52 metre.

**Step 4.** The RC yells the number of pins he has collected, including the then last one, for instance “three”. The HC checks the number of remaining pins he has in his hands. This number must be eleven, minus the number that the RC has yelled. In this case the HC number must be: $11 - 3 = 8$. The HC then yells “OK”. The total length measured along the line is in this example: $3 \times 30.00m + 4.52m$, which is equivalent to 94.52 metre.

**Step 5.** Next, the steps 1 to 4 should be repeated in the reverse direction. The result of the second chaining will not result in exactly 94.52 metres. It might, for instance, be 94.55. The difference is not allowed to exceed 1-2cm per tape length. Hence in the example, the difference has to be 5cm or less. In our chaining example the difference is an acceptable 3cm.

**Resulting length.** The average of the two measured lengths is 94.54m.

**Reducing slope length to horizontal length**

A horizontal length and a vertical length (height difference) form the perpendicular sides in a square triangle. According to the Pythagorean theorem, the length of the horizontal side is always shorter than the sloping side, see Sec. 3.1. Therefore a tape measure must be kept horizontal, which is only feasible for lengths up to about 10m. A length measured along a sloping surface needs to be reduced to the proper horizontal equivalent, see Fig. 34.

For small slope angles, however, reduction of length is not relevant, as Fig. 35 shows. Up to a slope angle of $2^\circ$ the required reduction does not exceed 2cm per full tape length (3,000cm). Even for a slope at a gradient of 1:10 (slope angle $5.7^\circ$) the reduction does not exceed 15cm on a full tape length (30m), which denotes a relative difference in length of 0.5%.
Figure 34: A slope length (S) must be ‘reduced’ to a horizontal length (H) by taking into account the gradient or the slope angle.

![Diagram](image)

The difference B′-B between slope length A-B and the equivalent horizontal length A-B′ depends on the size of the slope angle or gradient. In construction surveying the size of B′-B can be neglected completely for gradients smaller than 1:30 or angles smaller than 2, see Fig. 35

<table>
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<th>H (cm)</th>
<th>S (cm)</th>
<th>angle</th>
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<td>2999.9</td>
<td>3000</td>
<td>0.6</td>
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<td>1 : 50</td>
<td>60</td>
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<td>2985</td>
<td>3000</td>
<td>5.7</td>
</tr>
</tbody>
</table>

Figure 35: Gradients (expressed as V/H ratios) and the equivalent slope angles.

3.4 Applying square (90°) horizontal angles

Horizontal square angles are frequently used in construction surveying because they can be set out with quite simple tools such as:

- A tape measure, which is the simplest device for precisely constructing and checking square angles, see Fig. 36 & 37. Steel tape-measures must be bought.
- A **self-made cross-staff** (surveyor’s cross), which is easy to use but needs range poles for aiming. Results are less accurate than with a tape measure.
- An **optical square**, which is smaller and lighter than a cross; it therefore is easier to use, but must be bought.

![Figure 36: Two ways to construct a line perpendicular to a survey line.](image)

**Figure 36: Two ways to construct a line perpendicular to a survey line.**

Set pin P1 on line AB at C where a perpendicular has to be erected.
Set pin P2 on line AB at 3 metre from C. One surveyor holds the tape-measure with both the begin (0.00) and the 12.00 metre mark at pin P1.
The other surveyor wraps the tape around pin P2 and stretches the tape with pin P3. He sets P3 at the 8.00 metre mark.
Line P1 to P3 indicates the perpendicular direction to D. Extend the perpendicular beyond P3 and set point D at the required length from C.

**Figure 37: Erecting a perpendicular line from point C on survey line AB by constructing a rectangular triangle that fulfils the 3:4:5 rule.**
Setting out a line that is perpendicular to a survey line may be realised in two ways, see Fig. 36:
- **Erecting** the perpendicular *from the line* at some point on the line.
- **Dropping** the perpendicular *to the line* from a survey point outside the line.

**Using a tape-measure**
According to the Pythagorean theorem (Sec. 3.1), a triangle with a side ratio of 3:4:5 has a square angle between the two sides with length ratio 3:4. This geometric rule can be applied for setting out a perpendicular *from* a survey line.

1. Use pin P1 to fix the tape-measure at point C.
2. Put a second pin P2 on the survey line AB. An assistant is needed to align P2 with AB.
3. Stretch the tape between C and P2.
4. Take a third pin and hold this pin together with the tape between thumb and finger at P2.
5. Keep the tape stretched and describe a circle from P1 to the point at which the circle intersects with the survey line (to be indicated by the assistant).
6. Set a third pin P3 at that point on line AB.
7. Measure the length between P2 and P3 and put another pin, P4, at point D, exactly halfway between P2 and P3. This is the foot of the perpendicular from C to AB.

**Figure 38:** Dropping the perpendicular from point C to the survey line AB. (a) First construct an isosceles triangle with summit C and base P2-P3. (b) Then split the base of this triangle into two equal parts at point D. Line CD is the perpendicular to survey line AB.
Four length marks on the tape indicate three successive parts that have an exact length ratio of 3:5:4. These marks may be set, for instance, at 0.00, 3.00, 8.00 and 12.00 metre; or at 0.00, 6.00, 16.00 and 24.00 metre. Two persons can execute this 3-4-5 method, as shown in Fig. 37, but it is more convenient to have three persons, one at each corner. The 3-4-5 method is not very suitable for setting out a perpendicular towards a survey line from a point positioned outside this line. In this case, a method should be applied based on the construction of an isosceles triangle. The trick is to split the base of this triangle into two equal parts, see Fig. 38. The line from the top to the middle of the base is the required perpendicular. This method requires two persons.

**Figure 39: Self-made cross-staff (a) and view through a sight from both sides (b) and (c).**

**Using a self-made cross-staff**

Constructing a do-it-yourself cross-staff with two sights is easy. A wooden plate of about 40cm square and some nails is all one needs. A crucial requirement of a self-made cross-staff is that the two sights be mounted exactly at square angles. The best way to achieve this is by drawing diagonals in a precisely constructed square on the wooden plate, see Fig. 39a. The construction of a sight for a staff is basically identical to the sight of a rifle. The two sights can be made of ordinary nails: one at one end of a constructed diagonal (line of sight), and two nails at the other end,
see Fig 39a. Raised borders should be mounted on the sides of the plate to protect the nails from bending. These borders may also be made from plywood. Each sight must enable aiming from either side; see Fig. 39b & c.

Two precautions when using a cross-staff:
1. Keep the supporting pole vertical using a bull's-eye level (Fig. 12).
2. Prevent the cross-staff from rotating when changing the line of sight.

Though a cross-staff may be operated by one man, as illustrated in Fig. 41, it is much easier to have an assistant behind range pole A (or B) who instructs the surveyor in keeping the staff’s support aligned with A-B.

Using a cross-staff for erecting a perpendicular from a survey line is illustrated and explained in Fig. 41. This procedure requires two persons, one operating the cross-staff and instructing the other where to mark the end of the perpendicular.

When dropping a perpendicular to a survey line the working order is like that of erecting a perpendicular. In this case, however, a range pole is set at the correct position, D, from where the perpendicular DC
to line AB should start. The cross-staff is then manoeuvred into the correct position, C, on line A-B.

1. Put the cross-staff on survey line A-B at position C, where the perpendicular has to be erected.
2. Aim one sight of the cross-staff at the range pole A. Look through that same sight in the opposite direction to range pole B to check whether C is indeed on the line A-B. If not, check the alignment of A-C-B again.
3. An assistant puts a third range pole at the required length from point C. Use a tape-measure to set out this length. (Sec. 3.2.)
4. Change the view to the second sight while keeping the cross-staff in the same position. Do not rotate the staff.
5. Look through the second sight and instruct the assistant to go left or right until his range pole is exactly in the line of sight.
6. Go back to the other sight again and check whether the cross-staff is still properly aimed at point A or B. If not, then repeat steps 2-6.
7. Place the range pole and check the length from C to D. Use a pin or a peg to mark point D.

Figure 41: Erecting a perpendicular from a survey line A-B by means of a cross-staff.

When the staff is positioned on line A-B, the surveyor aims a sight at pole A (or B) and then looks through the other sight in the direction of pole D. Depending on his observation, he moves the cross-staff left or right (without rotating it) along line A-B until his sight points exactly at pole D. (The assistant meanwhile instructs him in keeping the staff
aligned with AB.) Most importantly, the surveyor must check con-
stantly whether the first sight remains aimed at pole A (or B).

**Using an optical square instead of a cross-staff**

Erecting or dropping a perpendicular can be executed more conven-
iently using a small and lightweight optical square, see Fig. 42.

![An optical square offers two aligned sights, to the left and right, in a single view (180-degree angle). Simultaneously it offers a third sight perpendicular (90-degree angle) to the aligned sights.](image1)

**Figure 42: Viewing through an optical square. The surveyor can look in three horizontal directions simultaneously.**

![First move the optical square forwards and backwards until the sights to the poles left and right are in line (Fig. 44b). This indicates the plumb-bob (C) to be aligned with A-B. When erecting a perpendicular, the front pole is moved left and right until seen in line with the sights left and right (Fig. 44c).](image2)

**Figure 43: The horizontal position of the optical square above the surface is indicated by means of a plumb-bob.**

Apart from easier handling, a major advantage of the optical square over the cross-staff is that the former enables observation of both lines
of sight simultaneously by means of two small prisms. Each prism deflects the line of sight over a 90° corner, one prism being aimed to the left and the other to the right. The two prisms are mounted between glass plates that allow for a forward line of sight. A plummet is used to position the optical square perpendicular above a line or a point, see Fig. 43.

![Diagram of optical square](image)

**Figure 44:** View through an optical square. (a) Plumb-bob not aligned with range poles marking the survey line. (b) Plumb-bob aligned with survey line, but either the front pole is not yet set to the perpendicular (erecting) or the bob is not yet above the foot of the perpendicular (dropping). (c) Both plumb-bob and front pole correctly positioned (aligned and perpendicular).

Training is required to use an optical square swiftly and accurately, but the procedure is comparable to that used with a cross-staff. *Dropping a perpendicular* to a survey line can be done in a one-man operation. *Erecting a perpendicular* from a point on a survey line, however, still requires the help of an assistant. In the latter case, the surveyor holds the plummet of the optical square above the point on the survey line from which the perpendicular has to be erected, instructing the assistant to move the front pole left or right.

**Accuracy of square angles**

In all cases, square angles are set in respect of an existing survey line. Without an optical aid, range poles can be aligned up to about 100m...
with a precision of about 2cm per 30m perpendicular to the line of sight. Square angles can be constructed with a tape measure (3-4-5 method) and will show precision comparable to that of chaining (1-2cm per 30m). With a an optical square, range poles used for aiming need to remain within a distance of 30m. Compared to the 3-4-5 method, precision is somewhat less (2-3cm per 30m), but the optical square is more flexible in use. The precision offered by a do-it-yourself cross-staff depends to a large extent on the way it has been constructed. With a thoroughly and rigidly built cross-staff, results can be expected comparable to those obtained with an optical square.

3.5 Dealing with non-square horizontal angles

Simple tools and problems of accuracy
One important aspect of angle measurements is relative accuracy. An angle error results in displacement lateral to the measured direction, which increases proportionally with distance. For instance, with a 1-degree error, the lateral displacement is 1.75 centimetre per metre, or 1.75 percent.

Figure 45: An angle error of only 1° causes lateral displacement equalling 1.75% of the distance from the point at which the angle is measured.

At a distance of 100 metres, this results in a 1.75-metre error, see Fig.45. Even at a distance of only 10m the effect is still 17.5cm, which is far too much with respect to the requirement of 1-2cm per 30m set for chaining, as mentioned at the end of Sec. 3.3.
When using a magnetic compass, the Magnetic North Pole provides a permanent reference direction. The clockwise horizontal angle (azimuth) from North to the aiming direction is read by means of a 360° compass card, see Fig. 16 in Sec. 3.1. Unfortunately, magnetic storms may cause temporal variations in the true geographic direction of Magnetic North. These variations can be as large as 1°. Local magnetic disturbances occur due to large steel objects, such as nearby cars, an iron bridge, or due to strong electric currents in a power line. These disturbances cause additional deviations to true Magnetic North. Therefore, *angles derived from measurements made with a compass must be regarded with suspicion.*

*Figure 46: Aiming and reading a hiking compass (explanation in the text).*

The kind of compass spoken of in this paragraph is normally used for hiking. It is supplied with a very simple sight. The magnetic needle pivots above a compass card that can be rotated by hand, see Fig. 46a. A mirror set at 45° above the compass card enables a simultaneous view through the sight and of the card. (Mirror is not shown in Fig. 46a). The gradation of the compass card of a hiking compass is generally 1-degree. A hiking compass is therefore not suited for accurate angular measurements. More accurate (and expensive!) compasses offer a compass card with 0.1° gradation.
Using a hiking compass
Due to its limited accuracy, the use of a hiking compass for survey work must be restricted to assessing the direction of a single reference line. This might be, for instance, a baseline needed for setting out a construction layout at a site.

When used for survey work, a compass should be placed on a support such as a cross-staff, even though a hiking compass is a hand-held device. For measuring the azimuth angle from Magnetic North to a target point, the sight of the compass must be aimed at that point, see Fig. 46b. The card must be rotated until the North and South marks coincide with the needle ends, while keeping the sight aimed at the target point, see Fig. 46c. When the card is set correctly, the azimuth angle can be read, see Fig. 46d, resulting in value 328°.

Figure 47: Calculation of the angle from a baseline to a centre line resulting from two azimuth angles measured with a hiking compass, (explanation in text).

For mapping purposes it is convenient to determine the angle between a baseline and another physically marked line, such as the centre line of a canal or that of a road. In such cases an azimuth angle must be measured for either of the two directions, as illustrated in Fig. 47. In Fig. 47a, two azimuth angles are measured. One angle is measured at point A, somewhere on the baseline SP1-SP2; the other angle at
point B, somewhere on a centre line. The measured clockwise azimuth angle from N to the baseline P1-P2 is 328°. The anti-clockwise angle is –32° (328° minus a full circle of 360°), see Fig 47a. The two azimuth angles from N for the two measured directions are 27°+32°, see Fig. 47b. The angle from the baseline P1-P2 (1) to the centre line (2) in Fig. 47c results from the calculation +27°– (–32°) = 27°+32° = 59°.

Using a surveying instrument
As stated previously, optical surveying instruments are beyond the scope of this Agrodok. The one exception is the use of a levelling instrument, which will be discussed in Chapter 4. Some levelling instruments, but definitely not all, are supplied with a horizontal circle for orientating survey lines on a building site. Some instruments have a 0.1° gradation. This gradation enables angle measurements of far better accuracy than can be achieved with a good-quality magnetic hiking compass.

![Diagram of measuring horizontal angles with a surveying instrument](image)

If direction A-B serves as the reference direction, then the angle from A-B to A-C is obtained by taking the difference between the two instrument readings for these directions: 102° – 67° = 35°

Figure 48: Measuring horizontal angles with a surveying instrument. One direction is chosen as the reference direction to determine angles to other directions.

Unlike a magnetic compass, a levelling instrument lacks any ‘built-in’ reference direction. Moreover, all surveying instruments are used without any fixed external reference for the direction in which the ‘zero’ of the horizontal circle points. Therefore the direction ‘0’ will point in a completely arbitrary direction, a feature inherent to all surveying instruments used for angle measurements with the exception of those with a built-in compass. Fortunately, this arbitrary orientation of the horizontal circle does not cause a problem. The orientation of such a horizontal circle is simply established by using one of the measured
Directions as the reference direction for all other measured directions, see Fig. 48. Such a reference direction is called a ‘back sight’ and its angular value from ‘zero’ is called the ‘orientation angle’.

### 3.6 Applying square (90°) vertical angles

Square angles with respect to the plumb-line are widely used when realising constructions like houses, schools, bridges or dams. Floors need to be laid level and walls have to be erected vertical. To achieve this, carpenters make use of a device called a ‘level’. Carpenter’s levels have been in existence since living memory. All technical realisations of the carpenter’s level are based on two methods of generating the reference direction:

- by means of a liquid, either in an open container (rare nowadays) or sealed in a transparent tube with a vapour bubble, as used in all modern carpenter’s levels;
- or by means of a plumb-bob on a string, known as the A-frame and in use since ancient times.

In this section we will discuss the modern carpenter’s level and the A-frame. Both types of levels can be set horizontal (or vertical), with an accuracy of about a millimetre per meter length. This is more than a hundred times less accurate than required for surveying purposes. Carpenter’s levels and A-frames are therefore NOT suited for precise levelling over distances exceeding several tens of meters, as is specifically required in the construction of most hydraulic structures. In Chapter 4 we discuss how to carry out accurate levelling by means of a levelling instrument.

**Using a modern carpenter’s level**

A modern carpenter’s level is made out of a rigid aluminium profile provided with at least two tubular bubbles, see Fig. 49. One bubble is for horizontal use of the level, the other for vertical use. These carpenter’s levels are available in lengths ranging from several decimetres to several metres. The longer have several horizontal and vertical bubbles.
Figure 49: Modern carpenter’s level supplied with two bubbles, one for horizontal use, the other for vertical use. Such levels are available in lengths ranging from several decimetres to several metres.

Check a carpenter’s level
The axis of a tubular bubble indicates a level plane. When the bubble is in the centred position (indicated by a gradation) the level is supposed to be exactly in a horizontal or vertical position, see Fig. 50a. Thus when the bubble is off centre the level is not in an exact horizontal or vertical position (Fig. 50b). This principle only holds if a bubble is correctly aligned to the longitudinal axis of the level (or to its perpendicular).

By using a level in two opposing positions, the alignment of a bubble can be checked, as shown and explained in Fig. 51a. Therefore, a level must always be used in two opposing positions. This method also enables a level to be used correctly even if the bubble proves to be defectively aligned, see Fig. 51b. Some carpenter’s levels allow for correction of bubble alignment.
Checking bubble alignment:
1. Place the level on a plane or beam that can gradually be tilted. Tilt the plane until the bubble is in its centred position, as indicated by the gradation.
2. Rotate the level into the opposite position. If the bubble does not return into its centred position then its axis is misaligned with the longitudinal axis of the level.

Using a defectively aligned bubble: tilt the plane supporting the level until the bubble returns to exactly the same gradation in both opposing positions of the level.

Figure 51: (a) Method used to check whether a bubble is correctly aligned. (b) Method for correct use of level when bubble proves to be defectively aligned.

1. Set peg A at the required height.
2. Use a beam to set peg B1 at a distance from peg A just shorter than the length of the beam. Set peg B1 higher than peg A.
3. Use a level to keep the beam level. Make a mark on peg B1 to indicate the correct height.
5-7. Repeat steps 2-4. Set peg C2 at the same height as peg A and B2.

Figure 52: Using a straight beam and carpenter’s level to stake successive pegs with their heads all at the same height.
Setting out pegs or hubs at equal heights
By using a beam with a length of about three metres, pegs or batter-boards can be set at the same height. The beam must be dead straight and rigid. The thickness may not vary more than a few millimetres over its full length.

1. The two legs and the cross-bar must be made out of light but inflexible material to give a rigid frame. The three joints linking the three elements must be entirely free from movement. Although rope may be used to tie them together, additional bolts or nails are needed to realise joints that are free of slack.
2. The string needs to be thin, even and flexible. A stone is perfectly suited to serve as the plumb-bob, but it needs to be sufficiently heavy to stretch the string tightly.
3. A level mark is set at the cross-bar using the alignment procedure described in Fig. 40.

Figure 53: Construction of a classic carpenter’s level or A-frame.

Using an A-frame (the classic carpenter’s level)
The construction of a classic carpenter’s level clarifies why it is called an A-frame. Though it is easy to construct an A-frame in a do-it-yourself fashion, some basic constructional conditions need to be met, see Fig. 53 for an illustration of these. Despite a difference in appearance, the A-frame is used in the same way as a modern carpenter’s level. An A-frame uses the plumb-line instead of a bubble for providing the vertical reference direction. Its use is restricted to horizontal levelling. The plumb-line allows for using an A-frame as a clinometer, as we will clarify in the next section (Sec. 3.7).
Setting-out contours with an A-frame

A large A-frame with a span of about two metres can be used for setting-out points of the same height in a field. Such points mark a line with a constant height: a contour. Due to the limited accuracy of the method, the length of the set-out line has itself to be limited to a hundred metres or so. Anyhow, it is strongly advisable to end a contour at a marked point known to be at the same height as that marking its be-

1. Draw a straight line on a strip of cardboard and fix this strip in the middle of the crossbar, see (a).
2. Put the A-frame on a flat surface that is more or less level.
3. When the plumb-bob has come to rest, draw a mark on the cardboard on either side of the string, see (b).
4. Rotate the A-frame into the opposing position with the legs in exactly in the same places again.
5. Repeat step 2, see (c). Now there are two sets of marks on the cardboard strip, see (d).
6. Lay the A-frame flat on the ground. Use a ruler or a pair of compasses to divide the distance between the two sets of marks exactly into exactly two equal parts; see (e).
7. Put the A-frame on a plane or beam that can gradually be tilted. Tilt the plane or beam until the string is in the centre position as marked on the strip.
8. Rotate the A-frame into the opposing position. If the string does not return to the centre position, repeat steps 2-8 until the string is centred in step 8.

Figure 54: Alignment procedure for determining the level mark of an A-frame.
Benchmarks signalling the begin and end points for each contour should be set out in the direction of the slope before setting out level contours with an A-frame. We will explain this approach in Sec. 3.7. Thus setting out a contour should start from a point set at the required elevation (benchmark) of that contour, see Fig. 55. Rotating the A-frame cancels out possible misalignment of the level mark after every two successive positions; moving along a contour from one position to the next, rotation is preferable to shift.

**Figure 55:** (a) Setting-out contours by means of an A-frame. (b) Rotating the frame.

### 3.7 Dealing with slope angles

Gradient and slope angle, presented at the end of Sec. 3.3, are the two quantities for a quantitative expression of slope. A gradient can be set out by means of a level and beam in the same way as a level line is set out, see Sec. 3.6. This method is, however, practicable over a length
up to ten metres or so. A clinometer is specifically designed to measure (or set out) a slope angle or gradient over a length of more than a few metres. In this section we discuss the use of both a level and a clinometer for setting out a gradient.

**Using a level to set out a gradient**

Both a modern carpenter’s level (bubble-level) and an A-frame can be used to set out a gradient. For a bubble-level, the procedure is nearly identical to the one illustrated in Sec 3.6, Fig. 52. When setting out a gradient, the main difference is that first the fall or rise over the length between two successive pegs must be calculated. Next, this rise (or fall) is set out vertically on a peg, as illustrated in Fig. 56.

![Figure 56: Setting out a specified gradient by means of a level and beam.](image)

Steps 1-3 are identical to setting out a level line, see Fig. 52.

4. Then measure length L1 and multiply by the required gradient (for instance 1:10) to calculate H1.
5. Make a second mark on peg B1 at distance H1 below the level mark (i.e. 1/10th of L1) and set peg B2 to that level.
6. Repeat steps 2-5 to set the next pegs.

The advantage of an A-frame over a bubble-level is that an A-frame can be calibrated for any required gradient. The calibration procedure is almost identical to the one illustrated in Fig. 54. The main difference when using an A-frame is that first a beam must be set precisely at the required gradient, as shown in Fig. 56. The A-frame is then calibrated in both positions to the gradient of this beam. Once the A-frame is calibrated to the gradient, pegs can be set directly, as with a level beam; see Fig. 56 and 57.
Using a clinometer to set out (or measure) a slope angle
Conceptually, a clinometer does not differ from a level, its construction being based either on a bubble or on a plumb-bob. The main difference is that a clinometer is specifically designed to set out or measure slope angles or gradients. A clinometer can be very helpful in measuring slope angles needed for reducing slope length. For an explanation of reduced slope length see Sec. 3.3.

An A-frame may for instance, be transformed into a clinometer by supplying the crossbar with a calibrated gradation in degrees (Fig. 58) or in gradients. With a string length of precisely one metre (1,000mm) from point of suspension to the gradation on the cross-bar, a two-
millimetre division between marks is equivalent to a gradient of 1:500 or an angle of about 0.1 degree. All the components needed to build a do-it-yourself clinometer are a stone, a simple plastic protractor with 1° gradation, and very thin and flexible string, see Fig. 59a. The longer the string, the better will be damping of oscillations. This simple device does not, however, allow aiming of the sight and simultaneous reading of the circle.

![Image of a do-it-yourself clinometer and a Breithaupt clinometer]

**Figure 59:** (a) Do-it-yourself clinometer made of a protractor. (b) Breithaupt clinometer.

A factory-built clinometer, as shown in Fig. 59b, is constructed according to the same principle and does enable simultaneous aiming and reading. Unfortunately, the price of a simple clinometer is in the same range as that of a good quality magnetic compass.

A clinometer must be aimed at a point at the same height above the ground as the clinometer; if it is not, the line of sight is not parallel to the slope. Consequently, the measured angle is not the slope angle, and is thus false. The requirement of the line of sight being parallel to the slope can easily be achieved using two range poles, each with a clear mark at eye-level. The height from tip to mark must be identical for both poles, as illustrated in Fig. 60a.

The measuring procedure is as follows. The person operating the clinometer keeps the instrument next to the mark on the range pole held in his other hand. He then aims at the mark on the other range pole.
held in front of him by his assistant. Finally, he reads off the angle (or gradient), see Fig. 60b. This procedure ensures that the measured angle is equal to the slope angle.

![Diagram of slope angle determination](image)

**Figure 60: Determination of slope angle using a clinometer.** (a) Two range poles or rods with marks at equal height are used for marking a line of sight parallel to the slope. (b) The measured angle of this line of sight is equal to the angle of the slope.
4 Levelling with an instrument

The subject of this chapter alone would justify a treatise the length of an entire booklet. The coming twenty pages therefore offer no more than the basic concepts (Sec. 4.1), including a description of common equipment (Sec. 4.2). We do not address specific applications, but briefly present some simple methods (Sec. 4.3), followed by procedures for error prevention (Sec. 4.4).

4.1 Concepts

A level plane is not a flat plane
The surface of a lake in calm weather provides a plane that is everywhere horizontal. Such an overall horizontal surface is called a ‘level’ plane. This surface is not a flat plane because it follows the earth-curvature, as does the water surface of an ocean. In relation to the size of a construction site, the difference between such a (curved) level plane and a (flat) horizontal plane is irrelevant. Height measurements over larger distances, however, do require the earth-curvature to be taken into account. Such advanced applications are not discussed in this booklet; here a ‘level’ plane is synonymous with a ‘flat and horizontal’ plane.

Instruments
A levelling instrument, like a carpenter’s level, is built around a device that provides a reference direction perpendicular to the plumb-line, as we will explain in the next section. A levelling instrument, however, has an optical sight for aiming over distances, which a carpenter’s level does not have. A levelling instrument is used to express difference between the elevation (level) of a point of which the level is already known and that of another point of which the level is not yet known. A measured difference between two elevations or levels is called a ‘height difference’. Though an optical level can be hand-held, it is generally mounted on a tripod, as shown in Fig. 61.
In this booklet we do not discuss hand-held levelling instruments because their use is rather limited. Neither do we discuss the so-called water-hose level, a type of level that is both difficult and limited in use.

**Levelling is based on differential measurements**

The above concept actually implies that in a levelling procedure the levels of two points are compared with reference to the horizontal line of sight of the instrument, resulting in a height difference. The easiest way to grasp this is again by using a ‘from-to’ approach. First the instrument is aimed at a staff positioned on the point with known height (benchmark), see Fig. 61a. The staff carries on its front side a centimetre or millimetre gradation, see Fig. 70 & 71.

![Figure 61: (a) Levelling starts with the instrument at an arbitrary point and the staff positioned at a point with known height (backsight). (b) The staff is then moved to a next point (foresight) leaving the instrument where it is.](image)

The vertical distance from the point on which the staff stands to the levelled line of sight is read at the position on the staff indicated by a horizontal cross hair in the sight’s optics. The staff is then moved to a second point, leaving the instrument where it is, see Fig. 61b. The measuring procedure is repeated and the second reading subtracted from the first. The difference between the two heights measured on the staff, i.e. the ‘rise minus fall’, expresses the difference in elevation (level) between the two points, see Fig. 62.
The calculation of the unknown level can be written in two ways:

starting at height HB:
\[ HB + dHb - dHf = HP \]
or starting at height HP:
\[ HP + dHf - dHb = HB \]

Both expressions can be rewritten:
\[ HB + dHb = HP + dHf = H.I. \]

The three levels HB, H.I. and HP are expressed as the heights from an artificial reference level that has ‘zero height’.

**Figure 62:** The height difference (diff.H) equals the ‘rise minus fall’ from the height of one point to that of the other via height H.I of the levelled line of sight.

### 4.2 Equipment

**Levelling the instrument**

Though telescopes may differ considerably in their optical details, the general design is as illustrated in Fig. 63. The telescope is mounted on its tripod by means of a levelling head or tribrach, which enables the telescope to rotate around a vertical axis. The head of the tripod cannot be set exactly level (truly horizontal). The tribrach therefore has three screws (some have four) to level the telescope on the tripod. A circular level (bull’s-eye) indicates whether the telescope is approximately levelled. After the telescope has been set level with the bull’s eye, the line of sight must be levelled precisely. This precise levelling can be achieved in two ways: using a tubular bubble or ‘automatically’, by means of a built-in optical compensator. We discuss both solutions below.
The eyepiece (1) is focused in the plane of the cross-hairs (2). With a knob and a gear-wheel (3) an intermediate lens (4) focuses the image of the telescope objective lens, also in the plane of the cross-hairs (2).

**Figure 63:** The 15X to 45X magnifying telescope enables reading the gradation on a staff at the line of sight, as indicated by cross hairs, see Fig. 61 & 62.

**Hand-levelled instruments**
The so-called ‘dumpy-level’ is the simplest levelling instrument of all. The tubular bubble of its telescope must be centred by means of the three tribrach-screws, which is a quite cumbersome procedure, illustrated and explained in Fig. 64.

Levelling of the telescope can be simplified by adding to the dumpy level a special tilting screw, see Fig. 65, which enables centring the tubular bubble. The tribrach screws are only used for approximately levelling the instrument in the same way as shown in Fig. 64.

The problem with all bubble-levels is this: if the surveyor forgets to level the telescope all measured heights will be false until the telescope is correctly levelled. Correct use of a bubble-level therefore requires a lot of training. The drill to be ‘hammered home’ goes: *Always level the telescope (line of sight) before reading the staff.*

Unfortunately, the error of a non-levelled telescope can only be detected in a ‘closed-loop’ procedure. The basic idea of a closed-loop is that a series of height measurements starts and ends at a point with a known height (i.e. at a benchmark). We explain closed-loop levelling in Sec. 4.3.
**Levelling a dumpy-level**

Begin with levelling the telescope approximately by turning the tribrach screws to centre the circular bubble, see (a). Then level precisely by using the tribrach screws again and observing the tubular bubble. The procedure for precise levelling is as follows:

1. Set telescope parallel to the imaginary line over two tribrach screws (1 & 2). Rotate these screws simultaneously in opposite directions until both bubbles are in the positions indicated in (b).

2. Revolve telescope over a quarter circle. Rotate the third screw (3) until bubbles are in the situation indicated in (c).

3. Revolve the telescope an additional quarter circle in the same direction as in step 2. Check whether the tubular bubble remains centred. If not, adjust by means of screws 1 & 2, revolve telescope in the reverse direction (see step 1) and check bubble again*).

4. Aim the telescope at the staff and check whether the tubular bubble is still centred. If not, use the screw nearest to the telescope to centre the bubble.

*) If the bubble cannot be centred, the bubble axis is not perpendicular to the vertical rotation axis. This does not indicate per se that the instrument is upset, but it is strongly advisable to check whether the bubble axis and the line of sight are indeed accurately aligned, see Sec 4.4.

**Figure 64: Dumpy-level. The operator must precisely level the telescope by means of all three tribrach-screws.**
Figure 65: A tilting-level is a dumpy-level with an additional tilting screw. The operator levels the instrument approximately by means of a circular bubble. Next he uses the tilting screw to precisely level the telescope using the tubular bubble.

Figure 66: The operator levels the instrument approximately by means of the circular bubble. An optical compensator inside the telescope precisely levels the line of sight without any interference from the operator.

Automatically levelled instruments
In many applications, a closed-loop procedure is either not applicable or not practical. A next best is to enable automatic levelling of the line of sight. This has been achieved by dispensing of the bubble altogether. Instead, an optical element inside the telescope levels the line of sight, see Fig. 66. Modern automatic levels are very robust and reliable and can be used under harsh conditions.
Automatic levels are especially useful for people who only occasionally use a levelling instrument, because the line of sight always remains level.

**Focusing eyepiece and telescope**

Before taking any reading the instrument needs to be correctly focussed. The focusing procedure comprises two *successive* phases: first focus the eyepiece, and thereafter focus the telescope; *never reverse the order!!*

Focusing of the instrument should be done directly after the instrument has been levelled approximately. Thus with a dumpy level or a tilting level, focusing takes place before the telescope is precisely levelled by means of the tubular level. An automatic level does not need this precise levelling.

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**Focusing must be done before reading the staff. This ‘drill’ holds for all types of levelling instruments.**

![Focusing eyepiece and telescope](image)

**Figure 67: First phase of focussing. The eyepiece must be focussed on the cross hairs whilst the image of the telescope is invisible (hand in front of telescope) or completely blurred (telescope out of focus).**

**First phase of focussing: the eyepiece**

- Turn the telescope completely out of focus by means of the focussing knob (Fig. 63, number 3), resulting in a completely blurred image.
- Turn the eyepiece at the back of the telescope in an anti-clockwise direction until it stops. By doing so, the cross hairs become invis-
ble. Wait for a few seconds and then slowly turn the eyepiece back in a clockwise direction until the cross hairs again become faintly visible (Fig. 67a).

- Then turn the eyepiece very slowly in a clockwise direction until you see the cross hairs sharp and crisp (b). Turn a bit further until they become slightly blurred again.
- Turn back very slowly (c) in the anti-clockwise direction until you see the hairs sharp and crisp again (d). Look at the gradation around the eyepiece and note the position of the index mark (e).

![Diagram of focusing steps](image)

**Figure 68: Second phase of focusing (after correctly focusing the eyepiece). The telescope must be focused on an object near to the staff. The image must be free of parallax, which means that when moving the head behind the eyepiece the cross hairs do not move in respect to the image of the telescope.**

**Second phase of focusing: the telescope**

- Aim the telescope at an object near to the staff, for instance at the person holding the staff. Check whether you see the cross hairs clearly and sharply. If not (Fig. 68a and b), repeat focusing the eyepiece.
- Aim at the person holding the staff. Turn the focusing knob on the telescope back and forth until you find the sharpest telescope image.
- Move your head behind the eyepiece. If you see the cross hairs ‘fixed’ to the image, the setting of the telescope is OK (Fig. 68d). However, if the cross hairs move slightly in respect of the image (Fig. 68c), then there is parallax (difference in sight) between the
image of the cross hairs (eyepiece) and the image of the telescope. In this case, you must improve the focus of the eyepiece.
➢ Aim the telescope at the staff, focus the image again and check once more for image parallax.

**Using and reading the staff**

When the height of a surface has to be established by a number of measurements from a single position, the staff must be placed directly on that surface, see Fig. 62. A support beneath the staff is not necessary. Moreover, as a staff has the ‘zero’ of its gradation at the bottom, see Fig. 70a, using a support would introduce a reading error equal to the height of the support.

![A firm support for the staff is needed for two reasons: (1) to mark an intermediate point, and (2) to prevent the staff from sinking into the ground.](image)

*Figure 69: Cast-iron support for staff, designed to be used at intermediate points during closed-loop measurements.*

**The need for firm support**

However, when closed-loop levelling is being performed, the instrument is moved from station to station, see Fig. 74. Each time the instrument is moved, the staff must remain at the same position and be turned towards the instrument’s next position. If the staff rests on an uneven or soft surface, turning the staff will result in its base being slightly altered in elevation. As the vertical accuracy of a closed-loop measurement is often a matter of millimetres, the slightest vertical movement of the staff is unwanted and must be prevented. Therefore the staff must be supported in all its positions. In the case of a closed-loop measurement, the use of a support will prevent height errors, as we will explain later in Sec. 4.3.

Benchmarks made out of large stones, pegs, iron tubes and the like (see Sec. 3.2) offer a sufficiently firm vertical support for the staff. On soft soil, a temporary support for the staff will be needed at intermediate points. Special cast-iron supports are purpose-made for this, see
Fig. 69. But a stone with a flat bottom and round top will do the trick just as well.

Figure 70: (a) Front sides of staffs showing various gradations. (b) Reverse side of a foldable staff. The bubble serves to check whether a staff is held vertically.

Staff gradation and orientation of view
The gradations on the front side of level-staffs differ quite considerably, as Fig. 70 shows. Though it is a tedious task, a staff with so-called E-gradation can be built in a do-it-yourself fashion.
All modern levelling instruments are equipped with an upright-viewing telescope. Many older and some current cheap instruments, however, feature an upside-down telescope image. These instruments will inevitably lead to problems when used by the inexperienced, as we explain in Fig. 71.

Reading the staff at three horizontal cross hairs
The field of view of a levelling instrument’s telescope comprises four hairs: a vertical cross hair, a horizontal cross hair (middle hair) and two shorter hairs equally spaced from the horizontal cross hair, see Fig. 71. The latter two hairs are the stadium hairs.
For levelling, the staff must be read at all three horizontal hairs in a way explained in Fig. 72, page 90. This results in:

1. The height (in mm) of the line of sight from the bottom of the staff; this is the average of all three readings.
2. The distance (in m) from instrument to staff, resulting from the difference between the two stadium-hair readings.

**Good levelling practices:**

*Before reading:* check whether the telescope is levelled, see Fig. 64, and check whether the cross hairs are viewed free of parallax, see Fig. 68.

*When reading:* do not touch the instrument or the tripod, to prevent disturbing the line of sight.

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**Figure 71: An upside-down viewing telescope requires an inverted staff and a drill.**

All level-staffs have the 'zero' of their gradation at the bottom irrespective of the kind of gradation.

An important point is that the numbers next to the gradation must be read in an upright position through the telescope.

As the image observed through a telescope may be orientated either upright or upside-down, the numbers on the staff must be orientated accordingly.

It takes a lot of training to correctly read an upside-down staff. Errors are very easily made as shown in this figure.

**Warning!!**

Someone who has not been thoroughly drilled in using it properly should not operate a levelling instrument equipped with an upside-down viewing telescope.
4.3 Levelling methods

Even the most common applications of a levelling instrument are too numerous to be included in this booklet. We therefore restrict our explanation to two levelling methods: one bridging short distances with the instrument positioned on a single station; the other levelling over longer distances, which requires the instrument to be placed at multiple stations. The latter method requires a so-called closed-loop to get reliable results.

**Levelling from a single station**
From a single position, levelling can be applied for measuring the height of points with respect to a benchmark height. Levelling is also used for setting out points at a specified height (marked by the heads of pegs). Points to be levelled (measured or set out) from a single sta-

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**Figure 72: Reading a staff by means of a horizontal cross hair and two stadium hairs.**

Reading the staff at three hairs:
1. Read the middle (cross-)hair (1404 mm).
2. Read the upper and lower (stadium) hairs (1641 & 1171). Add the readings (2812) and divide the sum by 2. Check the result (1406) with the middle hair reading (1404); the difference must be smaller than 1 (mm). If this condition is not met, then repeat all three readings until it is.
3. Subtract lower hair reading from upper hair reading (470) and divide by 10. The result is the length from instrument to staff in metres (47.0 m).
tion may be aligned (as for profiles and sections), may be distributed randomly over an area (as for contours) or systematically spaced (as for equalising a terrain). If no benchmark is available, a permanently marked point should be used as a benchmark. If the height of this point bears no relation to that of any other point, its height may be arbitrarily assigned. The value can be, for instance 100.000 m; but never use 0.000 m in order to prevent negative height values for other points.

The length of the section, or the size of the area covered, is restricted by the maximum distance that may be practically bridged with a levelling instrument, i.e. 60-80 metres over open and level terrain. Levelling, however, requires a clear line of sight at eye-level. Moreover, the staff must be in view at the height of the lowest cross hair. Hence, vegetation and height differences can severely reduce the range over which heights can be measured or set out.

**Setting out points at identical heights**

Using a levelling instrument points can be set out at the same level without using a levelling staff. A range pole or a simple wooden stick is all one needs. On this pole or stick a mark must be made that indicates the level of the line of sight with respect to the level of the benchmark.

For points to be set out at the level of the benchmark, the procedure for correctly setting the mark on the pole or stick is as follows.

- Position the range pole or stick on the benchmark indicating the level at which all points must be set out.
- Aim the instrument at the pole or stick and focus the telescope correctly, as illustrated in Fig. 68.
- The operator behind the instrument now instructs the person holding the pole or stick where the latter should mark the level of the line of sight (i.e. the middle hair as seen through the telescope).
- Mark this level by means of paint, coloured tape or an elastic band; the latter can be made from a piece of bicycle tube.
- Check whether this mark is clearly visible through the telescope and set at the correct height on the pole or stick.
For points to be set at identical levels differing from benchmark level, the procedure explained above needs some additional steps.

- Establish the difference between benchmark level and the level to be set out.
- If the level to be set out is below that of the benchmark, put a second mark on the pole or stick above the mark indicating the line of sight. (If the level to be set out is above the benchmark, put the second mark below that for the line of sight.)

A number of such marks can be put on a single stick, each indicating another level to be set out. Multiple markings, however, create a risk of identifying the wrong mark and causing an undetected blunder in the set out level. Therefore separate sticks should be used instead, one for each level and with a clear numerical indication of that level.

**Setting out a contour**

Contours can be set out more conveniently and precisely using a levelling instrument than by using a conventional A-frame. (Setting out a contour with an A-frame is shown in Fig. 55). The procedure requires a pole or a stick with a mark indicating the level to be set out with respect to the level of the benchmark. How to set the mark at the correct height on the stick was explained in the preceding paragraph. The surveyor behind the instrument instructs the person holding the pole to move ‘up’/‘down’ the slope (or ‘left’/‘right’). At regular intervals the pole holder uses a peg or a small stick to mark the contour.

**Cross-sections and randomly distributed points**

For more complicated levelling jobs than the ones discussed so far, a levelling-staff is an indispensable device. Such jobs include, for instance, measuring or setting out cross-sections, and measuring terrain heights of randomly distributed points.

The reason for using a staff instead of a marked pole or stick is that each point has its own arbitrary level with respect to the level of the benchmark. For each point the basic rule applies of differential measurement of two heights on the staff (‘rise-minus-fall’), as explained previously in Sec. 4.1, see Fig. 62.
Linking multiple stations
For covering larger areas a single station will not suffice. In this case levelling requires occupying multiple instrument stations, when all single-station measurements must be linked according to the same reference level. Thus from each station a measurement to at least one benchmark must be included. A benchmark is a marked point of which the elevation (reference level) is already known.

Figure 73: First three stations (A,B,C) of a series of measurements documented in Fig 76 & 77.

A closed-loop comprises a set of measurements from consecutive stations (A-I). Both the first backsight and the last foresight of this set must be to a benchmark. The sketch shows two kinds of closure: returning to the benchmark of beginning (BM103), and ending on another benchmark (BM102). Numbers refer to the example on p.87-88.

Figure 74: Sketch of the closed-loop measurements documented in Fig 76 & 77.

In many cases the requirement that a benchmark must be within range of each single station cannot be fulfilled. In these cases, intermediate benchmarks must be created by linking successive rise-and-fall measurements, as illustrated in Fig. 73 & 74. For creating these additional benchmarks, first a so-called ‘network’ of marked points must be lev-
elled. Thereafter these marked and levelled points can serve as benchmarks for single-station levelling.

Creating a network of benchmarks requires one or more series of successively linked backsight-foresight measurements. Such a series is called a loop. Each loop always must be ‘closed’. There are two ways of closing a loop.

**Closed loop with single benchmark**

The simplest way of closing a loop is by ending with the last foresight with the staff at the same location to which the first backsight was taken. In Fig. 74, this position could be any of the positions 1 to 9. The advantage of the closed-loop method is, that no benchmarks are required. The closed-loop method should also be applied when only one benchmark is available.

**Open loop with two benchmarks**

When a closed loop is not feasible, a second way of closing a series of linked backsight-foresight measurements is to use an open loop with two benchmarks. The backsight of the first station must be taken with the staff at one benchmark, whilst the foresight of the last station must be taken to the other. In Fig. 74 this is the case for the loop from position 1 (BM 103) via position 3 to position 6 (BM 102), as well as for the loop from position 6 via position 8 to position 1.

**Marking intermediate benchmarks**

Closed-loop and open-loop levelling enables the creation of additional benchmarks for single-point levelling from several different stations. These intermediate benchmarks must be marked, at least temporarily. Point markers, such as a wooden peg or an iron tube, are presented in Sec. 3.2.

**Example of closed-loop levelling (Fig. 76-77)**

*Always use a well-structured fill-in form*

One unquestionable prerequisite for maintaining overview and control over a closed-loop levelling procedure is a well-structured fill-in form.
First cycle of back- and foresight readings

I. Backsight

Step 1: Position instrument at station.
(First station is A.)
Level the instrument by means of the circular bubble see Sec. 4.2.
Aim the instrument at an object near the staff; focus the instrument;
remove cross-hair parallax.

Step 2: Put staff on backsight position.
(First backsight position is at BM 103).
Aim the instrument at the staff, focus telescope and critically check
the image for parallax; remove any remaining parallax.

Step 3: Precisely level line of sight.
For dumpy-level or tilting-level only, not for automatic instrument.
Use the tubular bubble for precisely levelling the telescope.

Step 4: Take backsight reading
(First backsight is from A to BM 103.)
Note the results on the fill-in form; see Fig. 76.
Results: 1,086 [mm] middle; and 1,272 & 901 [mm] stadium hairs.

Step 5: Check backsight reading.
Rise: \((1,272 + 901)/2 = 1,086.5\) [mm]; \textit{must equal} \(1,086 \pm 1\) [mm]
In the example the condition is fulfilled. In the case of this condition
not being fulfilled the backsight reading must be repeated and checked
until it is. Therefore:

\begin{center}
Staff must remain in backsight position until reading is OK.
\end{center}
Length of backsight:
\[1272 - 901 = 371 \text{ [mm]} \times 100 = 37.1 \text{ [m]}\]

Height of the telescope’s line-of-sight (H.I.):
\[33.313 \text{ [m]} \text{ (benchmark)} + 1086 \text{ [mm]} \text{ (rise)} = 34.399 \text{ [m]}\]

When backsight reading is OK, staff moves to foresight position.
Instrument must remain in same station.

II. Foresight

Step 6: Move staff to foresight position
(First foresight position is at 2.)
Whenever possible, length of foresight is taken nearly equal to length of preceding backsight. So balance both lengths by counting steps when moving staff from backsight position to next foresight position.

Step 7: Take foresight reading
(First foresight reading is from A to 2.)
Results: 1,783, 1,913 & 1,651 [mm].

Step 8: Check foresight reading.
Fall: \((1,913 + 1,651)/2 = 1,782 \text{ [mm]}\); must equal \(1,783 \pm 1 \text{ [mm]}\)

Staff must remain in foresight position until reading is OK.

Length of backsight:
\[1,913 - 1,651 = 262 \text{ [mm]} \times 100 = 26,200 \text{ [mm]} = 26.2 \text{ [m]}\]

Height of point 2:
\[34.399 \text{ [m]} \text{ (H.I.)} - 1,783 \text{ [mm]} \text{ (fall)} = 32.616 \text{ [m]}\]

Step 9: Calculate first ‘rise-minus-fall’.
Rise-minus-fall: \(1,086 - 1,783 \text{ [mm]} = 697 \text{ [mm]} = -0.697 \text{ [m]}\)
Notation for preventing negative values:
Substitute \(-0.697 \text{ [m]}\) by \(X9.303 \text{ [m]}, (-0.697 + 10.000)\).
Step 10: *Check balance of lengths.*
With 37.1 [m] & 26.2 [m], the lengths of back- and foresight show an average relative difference of about one third, which means that the two lengths are not sufficiently balanced. Compensation of this difference is required during future cycles. (The reason for this is explained at the end of this section.)

When foresight reading is OK, instrument moves to next station. Staff must remain in same position.

**Subsequent cycles of back- and foresight readings**
Move the instrument to the second station (B) and repeat the cycle of steps 1 to 10 with the backsight to position (2) and the foresight to position (3); see Fig. 73. Then move the instrument to station (C) for the next cycle. Repeat the procedure until the last cycle closes the loop, which in the example is at benchmark BM 103, see Fig. 74.

**After closure of loop**
When the levelling loop is closed on the (other) benchmark, *two conditions must be fulfilled:*

1. **The sum of the lengths of all backsight readings must equal that of all foresight readings within +/− 5% of the sum.** If this condition is fulfilled, all errors caused by upset alignment of the line of sight will be cancelled out. If not, a residual error inevitably remains in the sum of rises and falls.

2. **The benchmark height where the first backsight has been taken, plus the sum of all ‘rises’ and ‘falls’ must equal the benchmark height at which the last foresight has been taken;** see Fig. 77.

Referring to the first condition, checking an instrument for the alignment of its line of sight requires a quite simple test. The second condition can never be fulfilled exactly due to inherent measurement accuracy. The level of acceptance depends on the accuracy required and on the length of the loop.
1. Measuring the height difference between two pegs (or benchmarks) about 50m apart can check the alignment of the line of sight. Two stations (A and B) need to be occupied.

2. The effect of an upset line of sight will be cancelled out in the readings if the (sum of the) backsight length(s) equals that of the foresight(s), as the figure shows. Hence, the rise/fall obtained at station B can be used as a reference for the result obtained at station A.

3. For a well-adjusted instrument of average precision, the difference between the two results should not exceed 0.5mm per 10m, i.e. 2.5mm in this example; re-adjustment is not discussed here.

*Figure 75: A ‘two-peg test’ for checking the alignment of a levelling instrument.*
4.4 Error prevention and levelling accuracy

As explained in Sec. 4.2, a staff support is used at intermediate points between benchmarks to ensure that the staff remains at exactly the same elevation when the instrument is moved from one station to another. A support does not introduce a height error because its height is cancelled out, as can be seen by raising the support at point 2 in Fig. 73 by, for instance, 123mm. But ...

... never use a staff support on top of a benchmark ...

... because, the height of a support will not be cancelled out unless it is used in both the first backsight (on the 1st benchmark) and at the last foresight (on the 2nd benchmark), marking begin and end point of the loop. And ...

... if a peg, stone or tube is used for marking an intermediate benchmark in a closed or open loop levelling, then the levelling-staff must be placed on top of the marker, definitely not next to it or on a staff support.

The above is specifically relevant when applying loop levelling to establish the elevations of additional benchmarks that must be based on the same reference elevation. Such additional benchmarks are required for single-station levelling from several stations.

Regularly check an instrument's alignment (the two-peg test)
The line of sight of a levelling instrument must be aligned either parallel to the bubble axis (for bubble levels) or perpendicular to the plumb-line (for automatic levels). Whether this condition is fulfilled cannot be determined directly. Hence the alignment of an instrument must be regularly checked by means of a two-peg test, see Fig. 75 on opposite page.
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*1 lower stadia hair, not visible
*2 x (332-102)

Figure 76: Example of closed-loop levelling, see Fig. 74.

Simple construction surveying for rural applications
Levelling with an instrument 101

Figure 77: Continuation of Fig. 76.

**START at benchmark BM 103**

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<th>H.pt.</th>
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**1st backsight**

3 B.S. reading of staff: middle hair (1086 mm)
7 upper hair (1272); lower hair (901)
8 subtract B.S. readings: 1272 - 901 = 371 (mm)
add B.S. readings: 1272 + 901 = 2173
813 check sum: 2173/2 must equal 1086 (+/- 1) mm
516 calculate H.I.: 33.313 (m) + 1.086 (mm) = 24.399 (m)
811 calculate length of B.S. (in m): 371/10 = 37.1

**1st foresight**

4 F.S. reading of staff: middle hair in millimetres (1783 mm)
9 upper hair (1913); lower hair (1651)
10 subtract F.S. readings: 1913 - 1651 = 262
add F.S. readings: 1913 + 1651 = 3564
1014 check sum: 3564 should equal 2 x 1783 = 3566 (+/- 1)
516 calculate H of pt. 2: 34.399 (m) - 1783 (mm) = 33.616 (m)
10112 calculate length of F.S. (in m): 262/10 = 26.2

**1st rise/fall**

31413 calculate rise/fall: 1086 - 1783 (mm) = -0.697 (m) --> fall
14 if fall then add 10,000 (m): -0.697 + 10,000 = X9.303

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**END at benchmark BM 103**

6116 check sum 3 - sum 4 = H.pt. End - H.pt. Begin: 10733 - 10734 = -1 (mm) = 33.312 - 33.313 = -0.001 (m) --> OK
11112 check sum 11 - sum 12 < 5%: 296.1 - 289.5 = 6.6 < (5.6/290) x 100 = 2.3% --> OK
31414 check H.pt. End - H.pt. Begin = sum rise/fall from Begin to End: -0.001 (m) = -0.001 (m) --> OK
**Levelling accuracy largely depends on good practices**

Levelling from a single position easily enables a precision better than 10mm per 100m length (which is about the practical maximum range). Closed-loop levelling is meant to bridge longer distances with a higher accuracy. An in-depth accuracy evaluation of closed-loop levelling is, however, beyond the scope of this Agrodok. The basic problem is, that the accuracy of the closure is inversely related to the length of the loop, i.e. to the sum of fore- and backsight lengths. A usable rule of thumb is 1.5mm per set of backsight & foresight, up to a maximum length of 1 km.
5 Good surveying practices, a summary

Some ‘commandments’

Stick to procedures and practice them!

Recommendations presented in this last chapter are far from complete. Nevertheless, they clearly emphasise that good surveying practices are foremost a matter of personal dedication and sense of responsibility.

Without the right mentality, good survey work is doomed to remain an illusion.

- Plan all work related to a survey job, which includes three phases: (1) preparing fieldwork, (2) executing on-site measurements, and (3) processing measurement data, see Sec. 2.1 to 2.4.
- Precise and error-free measurements alone do not provide for accuracy. Include sufficient redundancy by supernumerary measurements to provide a truly accurate (i.e. reliable) geometric layout, see Sec. 2.5 and Sec. 3.4.
- Always check a level’s alignment before using it. This holds just as true for a carpenter’s level as it does for an optical level, see Sec. 3.6 and Sec. 4.4.
- Meticulously maintain all equipment. Do not put away any equipment unless clean and dry.

Note-keeping: fundamental and crucial

Never rely on your memory; make well-structured and trustworthy field notes!

Keeping field notes is the most important and fundamental activity in surveying; see Chap. 2. Note all relevant information immediately and exactly in a hard-jacket exercise book, not on stray sheets of paper. As
the notebook contains all data, it deserves to be handled with care. Keep your notebook in a safe place. Should it be lost or damaged, your survey-work will also be lost and may need to be repeated, at least partially.

Never erase or change a number once noted. Instead, completely cross out the wrong number; put the correct number above it. Therefore, ensure that line spacing is sufficient to include corrections numerals.

- Use a hard and sharp pencil because then notes are water-resistant and will not fade or smear. Do not use ballpoints (they smear) or roller-ball pens (they are not water-resistant).
- Do not economise on paper; use space for your notes. They need to be readable and well-structured for other people as well. Always perform all possible checks on your measurement data before proceeding to the next measurement or leaving the site.
- Preferably, use tidy forms to keep data and results well-organised and comprehensible for others.

**Chaining: document, keep aligned, and pull hard**

Measure each length at least twice to enable detection of any mistake or error.

Chaining with a tape requires a crew of two chainmen (‘Rear Chain’ and ‘Head Chain’) who use clear vocal communication for executing procedures and keeping notes.
- Keep well aligned, especially while chaining over a distance exceeding one tape length, see Sec. 3.2.
- Keep the measuring tape straight and under tension. Pull hard whenever in doubt about proper tension, see Sec. 3.3.
- Document unambiguously what is chained and how, see Sec. 3.3 and 3.4.
- Before leaving the site, check completeness of all measurements.
Square and non-square angles: check and double-check

| Use square angles whenever feasible or applicable. | Use non-square angles only when unavoidable. |

**Square angles**

**Horizontal:**
- Check a square angle between two perpendicular lines, see Sec. 3.4, by first constructing a triangle and then applying the Pythagorean theorem by means of the 3-4-5 rule, see Sec. 3.1, Fig. 18.

**Vertical:**
- When using a carpenter’s level for setting out a horizontal line (i.e. for an angle square to vertical), always check the alignment of its bubble, see Sec. 3.6, Fig. 51 and 54.

**Non-square angles**

An angle is the resulting difference between a direction (‘to’) and a reference direction (‘from’), i.e. ‘to-minus-from’. The direction of the angular rotation can be expressed in two opposite directions: either clockwise or anti-clockwise, see Sec. 3.1, Fig. 14. This holds for both horizontal angles and vertical angles, see Fig. 15 & 16.

**Horizontal, see Sec. 3.5:**
- When using a magnetic compass, the direction to magnetic North can serve as reference direction with angular value ‘zero’ (zero-direction), see Fig. 46.
- If not using magnetic North as the reference direction, the ‘zero’ points in an arbitrary direction, which cannot be used as a reference. Hence, the direction to some reference point must provide the reference direction, see Fig. 47.
- This reference direction can have any angular value, see Fig. 48.

**Vertical, see Sec. 3.1 & 3.7:**
- The reference direction is always defined with respect to the direction of gravity; i.e. with respect to the plumb-line, see Fig. 16.
- When the reference direction is Zenith, the zero-direction is opposite to the direction of gravity; in this case, a 90° angle expresses a horizontal direction, see Fig. 16.
When the reference direction is horizontal, the zero-direction is perpendicular to the direction of gravity; a direction above the horizon is ‘up’ (‘elevation’) with a ‘positive’ sign (direction to Zenith is +90°), whereas a direction below the horizon is ‘down’ (‘depression’) and carries a ‘negative’ sign, see Fig. 16.

When using a clinometer ensure that at both ends the line of sight (from clinometer to target) is at the same height above the surface or the benchmark(s), see Fig. 60.

**Levelling with an instrument: note, check, ensure & check**

*Any type of levelling*

Always ensure that the cross hairs are observed free of parallax with respect to the telescope’s sharp image of the staff, see Sec. 4.2, Fig. 67 & 68.

- Before reading the staff ensure that the line of sight is levelled, see Sec. 4.2.
- During a reading ensure that the staff is kept still and vertical.
- After a reading but before moving the staff to a subsequent position (or the instrument to a next station), first check whether half the sum of the stadium-hair readings equals the middle-hair reading, see Sec. 4.2, Fig. 72.

*Loop levelling*

Always begin and end on the same point (‘closed-loop’); or begin at one benchmark and end on another benchmark (‘open loop’), see Sec. 4.3, Fig. 74.

- At intermediate points in a loop always position the staff on a firm and rounded support; never use a support on a benchmark.
- Before ending a loop ensure that the sum of backsight lengths is equal to the sum of foresight lengths.
- *Before leaving the site, complete all checks presented in the example in Sec. 4.3, Fig. 76-77.*
Further reading

A source for information on simple surveying techniques at a level comparable to that of this Agrodok is the Agromisa publication: Practical for Basic Land Surveying and Irrigation, written by P.I Adriaanse and S.A.M.T. Povel, 1999, 150 pages. Educational Material nr. 11, ISBN 90-70857-29-4, priced at Euro 18.25.

Though the mathematical and physical basis of surveying is generic, its implementation in daily practice may differ considerably between various fields of application and survey ‘schools’.

Surveying is a discipline characterised by apparently differing ‘national flavours’ because the education of surveyors is focused nationally, if not regionally. A textbook on surveying will be available in every country or region where surveying is part of some curriculum. Such a textbook will address local practices, standards and terms.

For obvious reasons the content of this Agrodok is coloured by textbooks available in the Dutch language only. This bias, however, could be neutralised to a large extent by using both an American and a British reference text. The American source is a highly recommended textbook specifically addressing construction surveying: Construction Surveying and Layout: A Step-By-Step Field Engineering Methods Manual (Hardcover), by Wesley G. Crawford. The book is available via the Internet (Amazon.com) for a list price of US$ 70. Used books cost about US$ 25. ISBN: 0-9647421-1-X

Note:
For those who like to study surveying at a considerable higher level than this Agrodok offers, two extensive surveying manuals are freely available via the Internet:

The ‘Manual of Surveying Instructions – 1973’ at:
  http://www.cadastral.com/73manl-1.htm
  and at:
  http://www.tech.mtu.edu/dhsc/manual.html

The ‘Army manual FM 3-34.331 Topographic Surveying’ at:
  http://www.globalsecurity.org/military/library/policy/army/

Numerous Internet links to surveying methods & techniques can be found via The Encyclopaedia of Land Surveying Links at:
  http://hometown.aol.com/t52berg/pubpage.htm
and on the Land Surveyor Reference Page at:
  http://www.lsrp.com/
Useful addresses

The International Federation of Surveyors (FIG) is an international, non-government organisation whose purpose is to support international collaboration for the progress of surveying in all fields and applications, see at:

http://www.fig.net

On its website, FIG maintains a Surveying Education Database (SEDB) that currently (2005) contains information on more than 250 institutes and 450 surveying courses in more than 70 countries, see at:

http://www.fig.net/sedb/

The emphasis, however, is on education at an academic level.

Via the Internet many links to professional bodies and educational institutions are available at:

and at:

http://www.lsrp.com/
Glossary

accuracy
Degree of conformity to a standard or accepted value; comprises two aspects: precision and reliability.

adjustment of data
Provides a means of averaging random errors which occur in all measurements. A process used to remove inconsistencies in measured or computed data by means of redundancy in measurements or in a control network.

altitude
The vertical angle between the plane of the Horizon and the line to the object which is surveyed or set out.

angle
The difference in direction between two converging lines in either a vertical plane or a horizontal plane.

angle, azimuth
The clockwise horizontal angle between the direction of a line and a chosen reference direction.

angle, vertical
The angle of elevation above (‘plus’) or the angle of depression below (‘minus’) the Horizon.

backsight (BS)
Measuring angles: the sight taken on a point being used as the ‘from’ direction of the angle. Levelling: the sight taken on the level staff held on a point of known elevation to determine the height of instrument (H.I).

baseline
1) Triangulation: the side of one of a series of triangles which is established with great care and from which the lengths of the other sides are derived. 2) Construction: the line that is used as a reference...
line for measurement of \textit{lengths} and \textit{angles} for the layout of a construction.

\textbf{batter-boards} \quad \text{Boards set at the corners of a building for stretching wire or string marking the limits of construction.}

\textbf{bearing} \quad \text{The clockwise or anti-clockwise angle measured from \textit{North} or \textit{South} used to describe the \textit{direction} of a line.}

\textbf{benchmark} \quad \text{A relatively permanent object with a known \textit{elevation} used as a reference height for \textit{levelling}.}

\textbf{boning rod} \quad \text{Also called a \textit{sight rail}. Horizontal timber fastened at a specified height on a vertical stick or rod. Used to set out \textit{height} points along a certain \textit{slope} angle or \textit{gradient}.}

\textbf{bubble axis} \quad \text{The horizontal line tangent to the upper surface of a centred bubble in a tubular \textit{level}.}

\textbf{calibration} \quad \text{The process of comparing a device with a standard to correct or compensate for errors, or for purposes of record.}

\textbf{chaining or taping} \quad \text{The operation of measuring a \textit{distance} on the ground with a chain or tape. Chaining and taping are synonymous.}

\textbf{clinometer} \quad \text{An instrument for measuring \textit{angles} of \textit{slope}.}

\textbf{closed loop} \quad \text{A series of consecutive measurements that closes on the beginning point.}

\textbf{construction layout} \quad \text{A survey performed to locate designed structures on the ground.}

\textbf{contour} \quad \text{An imaginary line on a site plan that connects points of the same \textit{elevation}.}

\textbf{control network} \quad \text{Can be \textit{horizontal} and \textit{vertical}. A series of points connected by lengths and \textit{directions} (or \textit{heights}) that serve as a common framework for all points on the site.}
correction  
A value applied to a measurement to reduce the effect of errors.

cross hairs  
A set of wires or etched lines placed in a telescope and used for sighting purposes. See also stadium hairs.

cross-staff  
Simple instrument with the same function as an optical square; is also known as ‘survey cross’.

degree  
A unit of angular measurement equal to 1/360th of a circle. Also, a unit of measurement for temperature.

direction  
A course or line by to reach a destination.

distance  
The space between two things, expressed by an angle or a length.

elevation  
The vertical angle above the Horizon. With respect to levelling, the vertical distance (length) of a point above or below a reference height.

error  
The difference between an observed or computed value of a quantity and the ideal or true value derived either from a mathematical condition or a standard.

error (blunder or mistake)  
A large discrepancy in the true value of a measurement. Not a normal (random) measurement error, but the consequence of a mistake.

error (random)  
Error that is accidental in nature and always exists in all measurements. The larger an error, the less frequently it will occur.

error (systematic)  
Those errors that occur in the same magnitude and the same sign for each measurement. Can be eliminated either by mechanical operation of the instrument or by mathematical formula.
error of closure  
*Angles*: the amount by which the actual sum of a series of angles fails to equal the theoretically exact sum.  
*Azimuth*: The amount by which two values of the azimuth of a line, derived via different surveys or routes, fail to exactly equal each other.  
*Levelling*: the amount by which two values of the *elevation* of the same benchmark, derived via different surveys (or routes), fail exactly to equal each other.

field notes  
The permanent and detailed record made of field measurements and observations.

foresight (FS)  
*Angles*: the sight taken on the culminating (‘to’) *direction* of the angle.  
*Levelling*: the sight taken on the *level-staff* to determine the *elevation* of a point.

grade (gradient)  
The *slope* of a surface or of ground with a rise (or fall) expressed as a ratio to the *horizontal* distance.

height  
The dimension in the *direction* of the *Zenith*.

height of instrument  
(H.I.) In differential *levelling*: the elevation of the *line of sight* of the *telescope* above the *reference height*.  
(h.i.) When using an optical instrument: the height of the telescope above the *station*.

horizon  
An artificial *horizontal* plane.

horizontal  
Perpendicular to *vertical* (plumb-line) at a certain point. Note: a horizontal *plane* is NOT equivalent to a *level* surface, as the former has no curvature and the latter is curved. For distances of less than a few hundred metres, ‘horizontal’ and ‘level’
are practically equivalent (though not equal).

**hub**
A wooden stake with the same function as a *peg*, but larger.

**length**
The dimension of (a part of) a straight line expressed in some physical unit, such as a metre or a yard.

**level**
1) Synonymous with *horizontal*. 2) A tubular or circular device to indicate when a line, a plane or an instrument is set level or *vertical*. 3) A levelling instrument.

**level, rod-**
An accessory, i.e. a circular level, for use with a stadium-rod, *level-staff* or *range pole* to make certain of a *vertical* position.

**level-staff (or -rod)**
A graduated rod on which stadia intercepts can be read, used with a *level (instrument)* to measure: (1) the *height* of the *line of sight*; and (2) the *horizontal length* from the instrument to the point at which the rod is positioned.

**level surface**
A surface having all points at the same *elevation* and perpendicular to the (vertical) direction of gravity.

**levelling, differential**
The levelling process of determining the difference in *elevation* between two points.

**levelling, indirect**
Measuring *vertical angle* and *slope* distances to determine the difference in elevation between the instrument and a point.

**line of sight**
The line extending from an instrument along which distinct objects can be seen. The straight line between two points.

**map**
A paper representation, at a reduced scale, of the features on a part of the earth’s surface, i.e. a construction site.
mark or monument  A physical structure which marks the location of a survey point.

North (magnetic)  The reference direction when using a magnetic compass for angles.

optical square  Two (prismatic) mirrors placed at opposite angles of 45° to the sighting line makes possible positioning along a line between two visible points and simultaneous projection of a 90° angle from this line.

parallax  The apparent movement of the cross hairs caused by movement of the eye.

peg  A wooden pin, mostly with a rectangular cross-section of 3 to 4cm and a length of about 30cm, used to temporarily mark survey points; smaller than a hub.

plane  Surface without curvature, such that a straight line joining any two of its points lies wholly within it.

plotting (a map)  The transfer of survey data from field notes to paper.

plumb line; plumb-line  1) The vertical direction; a line perpendicular to a level plane. 2) String with a weight at the end, used to test whether something is vertical.

point  Synonymous with station.

precision  The closeness of one measurement to another as demonstrated by random errors; indicates the degree of refinement in the measuring process.

profile  The graphical representation of a line on the earth’s surface performed by levelling and by plotting.

profile levelling  The process of determining the elevation of a series of points along a defined line.

prolongation  The lengthening or extension of a line in the same direction.
range-pole  Also called sight-rod. A straight slender wood or metal rod of varying length, with a (metal) point; used as a sighting object for points along a line.

redundancy  In a geometric construction: the number of (superfluous) quantities above the minimum number of quantities theoretically required to realise the construction.

reliability  The sensitivity of a measuring process to non-random errors, such as blunders or mistakes; depends on the degree of redundancy in a geometric construction.

sight  A device for aiming an instrument such as a magnetic compass or a levelling instrument.

slope  (1) An inclined (natural) surface, an excavation or an embankment. (2) The angle at which a surface deviates from horizontal.

stadium hairs  Additional (horizontal) cross hairs in the optics of an instrument, such as a level. An interception between the stadium hairs on a (levelling) staff is calculated and used to determine the length from the instrument to the staff.

station  A point the location of which has been determined by surveying methods and which is usually marked on the ground.

surveying  The art and science of determining the relative position of points on, above or beneath the surface of the earth by measurement of angles, distances and elevations.

survey pin  A metal pin used for marking points when using a tape for chaining; used in bundles of 11 pins for chaining long lengths.
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
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<tbody>
<tr>
<td>tape, measuring</td>
<td>A ribbon of steel on which gradations are placed for measurement of distances; synonymous with ‘chain’.</td>
</tr>
<tr>
<td>topographic surveying</td>
<td>Measurements taken for locating objects and the elevation of points on the earth’s surface.</td>
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<tr>
<td>turning point</td>
<td>A temporary point the elevation of which is determined by differential levelling.</td>
</tr>
<tr>
<td>vertical</td>
<td>The direction in which the force of gravity acts (<em>plumb line</em>).</td>
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<tr>
<td>vertical angle</td>
<td>The angle measured up or down from the <em>Horizon</em>.</td>
</tr>
<tr>
<td>zenith</td>
<td>The <em>vertical</em> point above a given point on earth.</td>
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